Proof of the existence of prime number between successive squares

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Abstract

In the present paper, we prove that for any *n* natural number, in the $\lfloor n^2,(n+1)^2 \rfloor$ interval there are always exist *6k±1* type $g_1, g_2,..., g_t$ natural numbers. We give a closed formula to the *n*-dependent value *t* and on the $g_1, g_2,..., g_t$ natural numbers. After all, based on the *Dénes-type Complementary Prime-sieve theorem* (see [Dénes 2001]), we prove that among the $g_1, g_2,..., g_t$ natural numbers for any *n*, there are at least one prime number.

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THEOREM 1.
Let *n* be any natural number, then there always exist a *q* prime number on which it is fulfilled

(1) $$n^2 \langle q \langle (n+1)^2$$

Proof
Let denote $p_1, p_2,..., p_i,..., p_r$ the all prime numbers less than n^2

(2) $$p_1 \langle p_2 \langle ... \langle p_i \langle ... \langle p_r \langle n^2$$

Since $(n+1)^2-n^2=2n+1$ ($n \geq 2$) then in the $\lfloor n^2,(n+1)^2 \rfloor$ interval has at least one natural number in the form $g_x=6z \pm 1$, where $x=1,2,3,\ldots,t$ és $z=1,2,3,\ldots$ Search for the first of these type (g_1) numbers after n^2 (see Figure 1).

(3a) $$n = 2k \; and \; k = 3s \; (s = 1,2,3,...) \;\; \Rightarrow \;\; n^2 = (2 \cdot 3s)^2 = 6^2 \cdot s^2 \Rightarrow g_1 = n^2 + 1 \rightarrow (6z+1)$$

(3b) $$\begin{aligned} & n = 2k \; and \; k = 3s - 1 \; (s = 1,2,3,...) \;\; \Rightarrow \\ & \Rightarrow n^2 = (2(3s-1))^2 = 4(3s-1)^2 = 4(9s^2 - 6s + 1) = 36s^2 - 24s + 4 = \\ & = 6(6s^2 - 4s) + 4 \Rightarrow g_1 = n^2 + 1 \rightarrow (6z-1) \end{aligned}$$

(3c) $$\begin{aligned} & n = 2k \; and \; k = 3s - 2 \; (s = 1,2,3,...) \;\; \Rightarrow \\ & \Rightarrow n^2 = (2(3s-2))^2 = 4(3s-2)^2 = 4(9s^2 - 12s + 4) = 36s^2 - 48s + 16 = \\ & = 6(6s^2 - 8s + 2) + 4 \Rightarrow g_1 = n^2 + 1 \rightarrow (6z-1) \end{aligned}$$

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(3d) $n = 2k+1 \; and \; k = 3s \; (s = 1,2,3,\ldots) \;\; \Rightarrow \;\; n^2 = (2\cdot 3s+1)^2 = 36s^2 + 12s + 1 =$
$= 6(6s^2 + 2s) + 1 \Rightarrow g_1 = n^2 + 4 \rightarrow (6z-1)$

(3e) $n = 2k+1 \; and \; k = 3s-1 \; (s = 1,2,3,\ldots) \;\; \Rightarrow$
$\Rightarrow n^2 = (2(3s-1)+1)^2 = 4(3s-1)^2 + 4(3s-1) + 1 =$
$= 4(9s^2 - 6s + 1) + 12s - 4 + 1 = 36s^2 - 24s + 4 + 12s - 4 + 1 = 36s^2 - 12s + 1 =$
$= 6(6s^2 - 2s) + 1 \;\; \Rightarrow g_1 = n^2 + 4 \rightarrow (6z-1)$

(3f) $n = 2k+1 \; and \; k = 3s-2 \; (s = 1,2,3,\ldots) \;\; \Rightarrow$
$\Rightarrow n^2 = (2(3s-2)+1)^2 = 4(3s-2)^2 + 4(3s-2) + 1 =$
$= 4(9s^2 - 12s + 4) + 12s - 8 + 1 = 36s^2 - 48s + 16 + 12s - 8 + 1 = 36s^2 - 36s + 6 + 3 =$
$= 6(6s^2 - 6s + 1) + 3 \;\; \Rightarrow g_1 = n^2 + 2 \rightarrow (6z-1)$

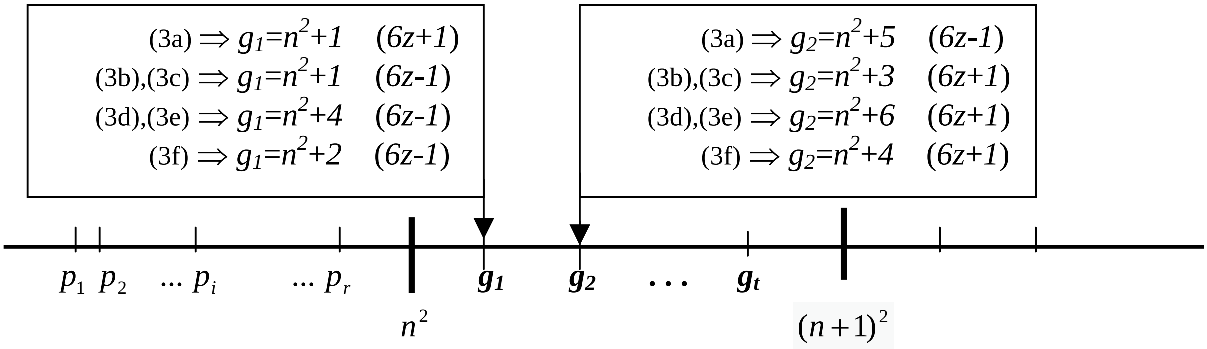

Figure 1.

According to the (3a)-(3f) formulas, g_1 is *6z+1 type* only in the case (3a), in all other cases are there of *6z-1 type*. Since we search the g_x=*6z±1* type natural numbers in the $\left[n^2,(n+1)^2\right]$ interval, then there exists in each case a g_2 number that is opposite type to g_1. So on g_2 are true of the following relationships:

(4a) $\overset{(3a)}{\Rightarrow} \;\; g_1 = n^2 + 1 \rightarrow (6z+1) \Rightarrow g_2 \rightarrow (6z-1) \Rightarrow g_2 = g_1 + 4 = n^2 + 5$

(4b) $\overset{(3b)}{\Rightarrow} \;\; g_1 = n^2 + 1 \rightarrow (6z-1) \Rightarrow g_2 \rightarrow (6z+1) \Rightarrow g_2 = g_1 + 2 = n^2 + 3$

(4c) $\overset{(3c)}{\Rightarrow} \;\; g_1 = n^2 + 1 \rightarrow (6z-1) \Rightarrow g_2 \rightarrow (6z+1) \Rightarrow g_2 = g_1 + 2 = n^2 + 3$

(4d) $\overset{(3d)}{\Rightarrow} \;\; g_1 = n^2 + 4 \rightarrow (6z-1) \Rightarrow g_2 \rightarrow (6z+1) \Rightarrow g_2 = g_1 + 2 = n^2 + 6$

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(4e) $$\overset{(3e)}{\Rightarrow} \quad g_1 = n^2 + 4 \to (6z-1) \Rightarrow g_2 \to (6z+1) \Rightarrow g_2 = g_1 + 2 = n^2 + 6$$

(4f) $$\overset{(3f)}{\Rightarrow} \quad g_1 = n^2 + 2 \to (6z-1) \Rightarrow g_2 \to (6z+1) \Rightarrow g_2 = g_1 + 2 = n^2 + 4$$

Table 1. presents some examples to calculating g_1 and g_2 values. Column 4 of the Table 1. shows the calculation of g_1 and column 7 shows the calculation of g_2 with the formulas (3a)-(3f) and (4a)-(4f). Columns 6 and 9 contain the values g_1 and g_2 (***bold highlighting*** indicates that g_1 or g_2 is prime).

We call g_1-type the next arithmetic series in the $\left[n^2,(n+1)^2\right]$ interval, and denote t_1 the number of elements of the series:

(5) $$g_1,\ g_1+6,\ g_1+2\cdot 6,\ \ldots,\ g_1+(t_1-1)\cdot 6$$

Using the formulas (3a)-(3f) we can determine the number of elements of the g_1-type series (t_1).

(6a) $$\overset{(3a)}{\Rightarrow} \quad g_1 = n^2 + 1 \Rightarrow$$
$$\Rightarrow t_1 = \left[\frac{(n+1)^2 - (n^2+1)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 1 + 3}{6}\right] = \left[\frac{2n+3}{6}\right]$$

(6b) $$\overset{(3b)}{\Rightarrow} \quad g_1 = n^2 + 1 \Rightarrow$$
$$\Rightarrow t_1 = \left[\frac{(n+1)^2 - (n^2+1)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 1 + 3}{6}\right] = \left[\frac{2n+3}{6}\right]$$

(6c) $$\overset{(3c)}{\Rightarrow} \quad g_1 = n^2 + 1 \Rightarrow$$
$$\Rightarrow t_1 = \left[\frac{(n+1)^2 - (n^2+1)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 1 + 3}{6}\right] = \left[\frac{2n+3}{6}\right]$$

(6d) $$\overset{(3d)}{\Rightarrow} \quad g_1 = n^2 + 4 \Rightarrow$$
$$\Rightarrow t_1 = \left[\frac{(n+1)^2 - (n^2+4)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 4 + 3}{6}\right] = \left[\frac{n}{3}\right]$$

(6e) $$\overset{(3e)}{\Rightarrow} \quad g_1 = n^2 + 4 \Rightarrow$$
$$\Rightarrow t_1 = \left[\frac{(n+1)^2 - (n^2+4)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 4 + 3}{6}\right] = \left[\frac{n}{3}\right]$$

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(6f)
$$\overset{(3f)}{\Rightarrow} \quad g_1 = n^2 + 2 \Rightarrow$$
$$\Rightarrow t_1 = \left[\frac{(n+1)^2 - (n^2+2)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 2 + 3}{6}\right] = \left[\frac{2n+2}{6}\right] = \left[\frac{n+1}{3}\right]$$

We call g_2-type the next arithmetic series in the $\left[n^2, (n+1)^2\right]$ interval, and denote t_2 the number of elements of the series:

(7) $g_2, g_2+6, g_2+2\cdot 6, \ldots, g_2+(t_2-1)\cdot 6$

Using the formulas (4a)-(4f) we can determine the number of elements of the g_2-type series (t_2).

(8a)
$$\overset{(4a)}{\Rightarrow} \quad g_2 = n^2 + 5 \Rightarrow$$
$$\Rightarrow t_2 = \left[\frac{(n+1)^2 - (n^2+5)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 5 + 3}{6}\right] = \left[\frac{2n-1}{6}\right]$$

(8b)
$$\overset{(4b)}{\Rightarrow} \quad g_2 = n^2 + 3 \Rightarrow$$
$$\Rightarrow t_2 = \left[\frac{(n+1)^2 - (n^2+3)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 3 + 3}{6}\right] = \left[\frac{2n+1}{6}\right]$$

(8c)
$$\overset{(4c)}{\Rightarrow} \quad g_2 = n^2 + 3 \Rightarrow$$
$$\Rightarrow t_2 = \left[\frac{(n+1)^2 - (n^2+3)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 3 + 3}{6}\right] = \left[\frac{2n+1}{6}\right]$$

(8d)
$$\overset{(4d)}{\Rightarrow} \quad g_2 = n^2 + 6 \Rightarrow$$
$$\Rightarrow t_2 = \left[\frac{(n+1)^2 - (n^2+6)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 6 + 3}{6}\right] = \left[\frac{n-1}{3}\right]$$

(8e)
$$\overset{(4e)}{\Rightarrow} \quad g_2 = n^2 + 6 \Rightarrow$$
$$\Rightarrow t_2 = \left[\frac{(n+1)^2 - (n^2+6)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 6 + 3}{6}\right] = \left[\frac{n-1}{3}\right]$$

(8f)
$$\overset{(4f)}{\Rightarrow} \quad g_2 = n^2 + 4 \Rightarrow$$
$$\Rightarrow t_2 = \left[\frac{(n+1)^2 - (n^2+4)}{6} + \frac{1}{2}\right] = \left[\frac{n^2 + 2n + 1 - n^2 - 4 + 3}{6}\right] = \left[\frac{n}{3}\right]$$

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We are obtained the number t of the g_x=$6z\pm1$-type elements in the $\lfloor n^2,(n+1)^2 \rfloor$ interval (see figure 1.) by summing the pairwise t_1 and t_2 (see (6a)-(6f), (8a)-(8f) and Table 1. column 10).

(9a) $$\overset{(6a),(8a)}{\Rightarrow} \quad t = t_1 + t_2 = \left[\frac{2n+3}{6}\right] + \left[\frac{2n-1}{6}\right] = \left[\frac{2n+1}{3}\right]$$

(9b) $$\overset{(6b),(8b)}{\Rightarrow} \quad t = t_1 + t_2 = \left[\frac{2n+3}{6}\right] + \left[\frac{2n+1}{6}\right] = \left[\frac{2n+2}{3}\right]$$

(9c) $$\overset{(6c),(8c)}{\Rightarrow} \quad t = t_1 + t_2 = \left[\frac{2n+3}{6}\right] + \left[\frac{2n+1}{6}\right] = \left[\frac{2n+2}{3}\right]$$

(9d) $$\overset{(6d),(8d)}{\Rightarrow} \quad t = t_1 + t_2 = \left[\frac{n}{3}\right] + \left[\frac{n-1}{3}\right] = \left[\frac{2n-1}{3}\right]$$

(9e) $$\overset{(6e),(8e)}{\Rightarrow} \quad t = t_1 + t_2 = \left[\frac{n}{3}\right] + \left[\frac{n-1}{3}\right] = \left[\frac{2n-1}{3}\right]$$

(9f) $$\overset{(6f),(8f)}{\Rightarrow} \quad t = t_1 + t_2 = \left[\frac{n+1}{3}\right] + \left[\frac{n}{3}\right] = \left[\frac{2n+1}{3}\right]$$

It follows from the above and from the Theorem 1. in [Dénes 2001] that there are always t pieces of $6z\pm1$-type $g_1,\ldots,g_t$ values (see (9a)-(9f)) in the $\lfloor n^2,(n+1)^2 \rfloor$ interval and only among them there exists the prime numbers in the intervals. Cases $n=3,4,\ldots,10$ are shown in Figures 2-9. In the Figures the arrows indicate the corresponding g_i values, with the **bold arrows** marked if g_i is a prime number.

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Examples

$$n=3 \overset{(9f)}{\Rightarrow} t=\left[\frac{2\cdot 3+1}{3}\right]=2$$

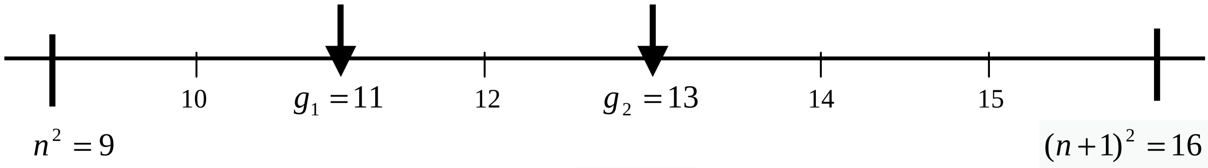

Figure 2.

$$n=4 \overset{(9b)}{\Rightarrow} t=\left[\frac{2\cdot 4+2}{3}\right]=3$$

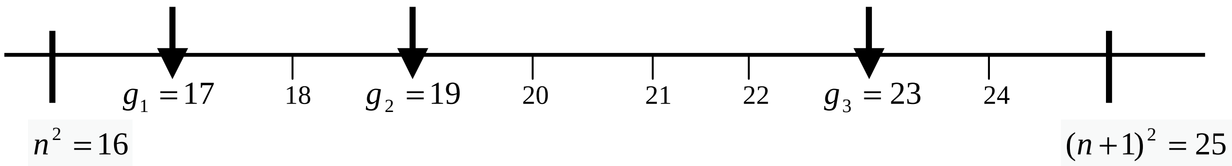

Figure 3.

$$n=5 \overset{(9e)}{\Rightarrow} t=\left[\frac{2\cdot 5-1}{3}\right]=3$$

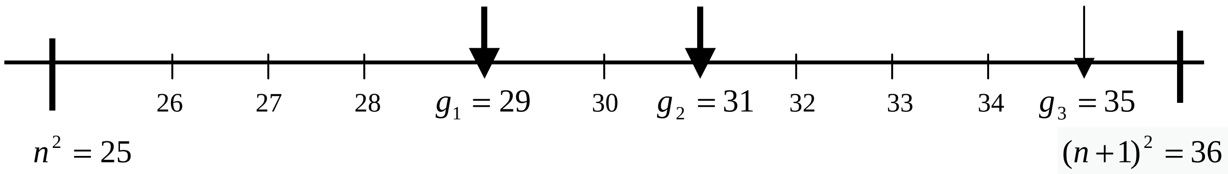

Figure 4.

$$n=6 \overset{(9a)}{\Rightarrow} t=\left[\frac{2\cdot 6+1}{3}\right]=4$$

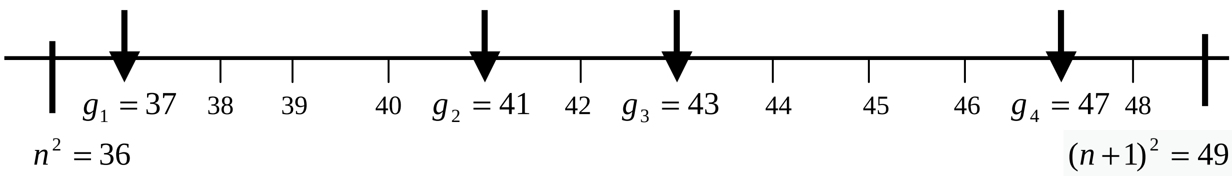

Figure 5.

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$$n = 7 \overset{(9d)}{\Rightarrow} t = \left[\frac{2 \cdot 7 - 1}{3}\right] = 4$$

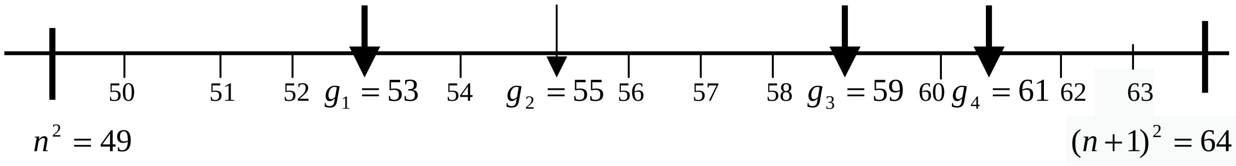

Figure 6.

$$n = 8 \overset{(9c)}{\Rightarrow} t = \left[\frac{2 \cdot 8 + 2}{3}\right] = 6$$

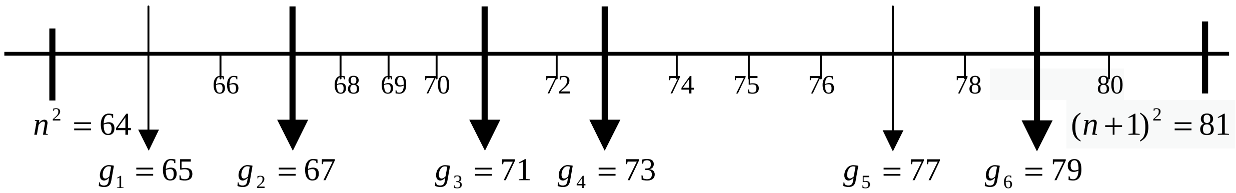

Figure 7.

$$n = 9 \overset{(9f)}{\Rightarrow} t = \left[\frac{2 \cdot 9 + 1}{3}\right] = 6$$

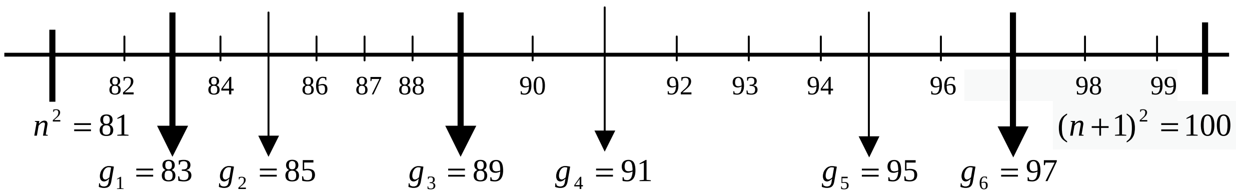

Figure 8.

$$n = 10 \overset{(9b)}{\Rightarrow} t = \left[\frac{2 \cdot 10 + 2}{3}\right] = 7$$

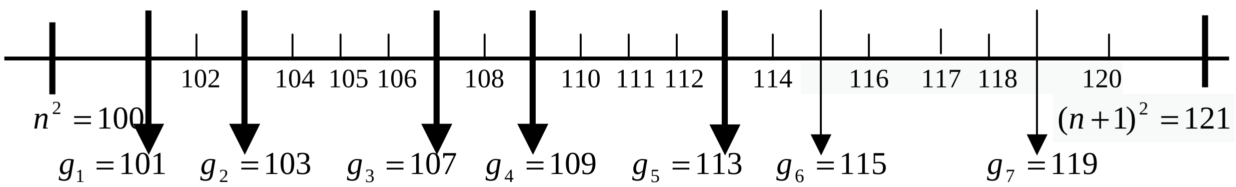

Figure 9.

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Continue of the proof **(indirect)**

As an indirect condition contrary to statement (1), let us suppose that the $g_1,...,g_t$-types values (see (5), (7)) for any natural number *n* is not a prime number. Therefore, based on the formulas (3a)-(3f), due to [Dénes 2001] Theorem 2, to each g_x ($x=1,2,\dots,t$) must be true one of the following equations:

(10a) $\forall n \text{ és } g_x = 6z+1 \;\; (x=1,2,...,t) \;\; \Rightarrow \;\; z = 6uv+u+v \;\; (u,v=1,2,3,...)$

(10b) $\forall n \text{ és } g_x = 6z+1 \;\; (x=1,2,...,t) \;\; \Rightarrow \;\; z = 6uv-u-v \;\; (u,v=1,2,3,...)$

(10c) $\forall n \text{ és } g_x = 6z-1 \;\; (x=1,2,...,t) \;\; \Rightarrow \;\; z = 6uv+u-v \;\; (u,v=1,2,3,...)$

(10d) $\forall n \text{ és } g_x = 6z-1 \;\; (x=1,2,...,t) \;\; \Rightarrow \;\; z = 6uv-u+v \;\; (u,v=1,2,3,...)$

We show that in the case of each g_x values ($1 \le x \le t$) for all *n* natural numbers (see (3a)-(3f) and (5), (7), as well as Figure 1.), the indirect statement always leads to a contradiction.

Due to the formulas (5) and (7) the values g_x ($x=1,2,\dots,t$) can be sorted into the following series:

(11)
$$g_1, g_2, g_3 = g_1+6, g_4 = g_2+6, ..., g_x = g_1 + \left[\frac{x-1}{2}\right]6, g_{x+1} = g_2 + \left[\frac{x-2}{2}\right]6, ..., g_t \Rightarrow$$
$$\Rightarrow g_1, g_2, g_3 = g_1+6, g_4 = g_2+6, ..., g_x = g_1+3(x-1), g_{x+1} = g_2+3(x-2), ..., g_t$$

For the rest of the proof, we will calculate all the pairs of formulas (3a)-(3f) and (10a)-(10d), which produce all *n* natural numbers (this is $6 \cdot 4 = 24$ cases). In other words, we produce the formulas to the g_x and g_{x+1} elements of the $g_1,...,g_t$ series due to the (10a)-(10d) correspondence.

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Case of the (3a) - (10a) – (10c):

(12a)
$$g_x = g_1 + 3(x-1) \overset{(3a)}{=} n^2 + 1 + 3(x-1) \overset{(10a)}{=} 6(6uv + u + v) + 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3a)}{=} n^2 + 5 + 3(x-2) \overset{(10c)}{=} 6(6uv + u - v) - 1$$

$$\overset{(12a)}{\Rightarrow} \frac{n^2 + 3(x-1)}{6} = v(6u+1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) - 6u}{6(6u+1)} \overset{(12a)}{\Rightarrow}$$
$$\overset{(12a)}{\Rightarrow} \frac{n^2 + 6 + 3(x-2)}{6} = 6uv + u - v \quad \Rightarrow$$
$$\Rightarrow \frac{n^2 + 6 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) - 6u}{6(6u+1)} + u - \frac{n^2 + 3(x-1) - 6u}{6(6u+1)} =$$
(12b)
$$= \frac{6un^2 + 18u(x-1) - 36u^2 + 36u^2 + 6u - n^2 - 3(x-1) + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 12u - n^2 - 3(x-1)}{6(6u+1)} \quad \Rightarrow$$
$$\Rightarrow \left(n^2 + 6 + 3(x-2)\right)(6u+1) = 6un^2 + 18u(x-1) + 12u - n^2 - 3(x-1) \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux + n^2 + 3x = 6un^2 + 18ux - 6u - n^2 - 3x + 3 \quad \Rightarrow$$
$$\Rightarrow \quad 2n^2 + 6x + 6u - 3 = 0 \quad \Rightarrow \quad u = \frac{-2n^2 - 6x + 3}{6} \langle 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (12a)-(12b) is contrary!

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Case of the (3a) - (10a) – (10d):

(13a)
$$g_x = g_1 + 3(x-1) \overset{(3a)}{=} n^2 + 1 + 3(x-1) \overset{(10a)}{=} 6(6uv + u + v) + 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3a)}{=} n^2 + 5 + 3(x-2) \overset{(10d)}{=} 6(6uv - u + v) - 1$$

$$\overset{(13a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1)}{6} = v(6u+1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) - 6u}{6(6u+1)} \quad \overset{(13a)}{\Rightarrow}$$

$$\overset{(13a)}{\Rightarrow} \quad \frac{n^2 + 6 + 3(x-2)}{6} = 6uv - u + v \quad \Rightarrow$$

$$\Rightarrow \quad \frac{n^2 + 6 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) - 6u}{6(6u+1)} - u + \frac{n^2 + 3(x-1) - 6u}{6(6u+1)} =$$

$$= \frac{6un^2 + 18u(x-1) - 36u^2 - 36u^2 - 6u + n^2 + 3(x-1) - 6u}{6(6u+1)} =$$

(13b)
$$= \frac{6un^2 + 18u(x-1) - 72u^2 - 12u + n^2 + 3(x-1)}{6(6u+1)} \quad \Rightarrow$$

$$\Rightarrow \quad \left(n^2 + 6 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux - 30u - 72u^2 + n^2 + 3x - 3 \quad \Rightarrow$$

$$\Rightarrow \quad 6un^2 + 18ux + n^2 + 3x = 6un^2 + 18ux - 30u - 72u^2 + n^2 + 3x - 3 \quad \Rightarrow$$

$$\Rightarrow \quad 72u^2 + 30u + 3 = 0 \quad \Rightarrow \quad 24u^2 + 10u + 1 = 0 \quad \Rightarrow$$

$$\Rightarrow \quad u_{1,2} = \frac{-10 \pm \sqrt{100 - 96}}{48} = \frac{-10 \pm 2}{48} \langle \, 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (13a)-(13b) is contrary!

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Case of the (3a) - (10b) – (10c):

(14a)
$$g_x = g_1 + 3(x-1) \overset{(3a)}{=} n^2 + 1 + 3(x-1) \overset{(10b)}{=} 6(6uv - u - v) + 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3a)}{=} n^2 + 5 + 3(x-2) \overset{(10c)}{=} 6(6uv + u - v) - 1$$

$$\overset{(14a)}{\Rightarrow} \frac{n^2 + 3(x-1)}{6} = v(6u-1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 6u}{6(6u-1)} \overset{(14a)}{\Rightarrow}$$

$$\overset{(14a)}{\Rightarrow} \frac{n^2 + 6 + 3(x-2)}{6} = 6uv + u - v \quad \Rightarrow$$

$$\Rightarrow \frac{n^2 + 6 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 6u}{6(6u-1)} + u - \frac{n^2 + 3(x-1) + 6u}{6(6u-1)} =$$

$$= \frac{6un^2 + 18u(x-1) + 36u^2 + 36u^2 - 6u - n^2 - 3(x-1) - 6u}{6(6u-1)} =$$

(14b)
$$= \frac{6un^2 + 18u(x-1) + 72u^2 - 12u - n^2 - 3(x-1)}{6(6u-1)} \quad \Rightarrow$$

$$\Rightarrow \quad \left(n^2 + 6 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux - 30u + 72u^2 - n^2 - 3x + 3 \quad \Rightarrow$$

$$\Rightarrow \quad 6un^2 + 18ux - n^2 - 3x = 6un^2 + 18ux - 30u + 72u^2 - n^2 - 3x + 3 \quad \Rightarrow$$

$$\Rightarrow \quad 72u^2 - 30u + 3 = 0 \quad \Rightarrow \quad 24u^2 - 10u + 1 = 0 \quad \Rightarrow$$

$$\Rightarrow \quad u_{1,2} = \frac{10 \pm \sqrt{100-96}}{48} = \frac{10 \pm 2}{48} \langle 1$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (14a)-(14b) is contrary!

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Case of the (3a) - (10b) – (10d):

(15a)
$$g_x = g_1 + 3(x-1) \overset{(3a)}{=} n^2 + 1 + 3(x-1) \overset{(10b)}{=} 6(6uv - u - v) + 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3a)}{=} n^2 + 5 + 3(x-2) \overset{(10d)}{=} 6(6uv - u + v) - 1$$

$$\overset{(15a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1)}{6} = v(6u-1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 6u}{6(6u-1)} \quad \overset{(15a)}{\Rightarrow}$$
$$\overset{(15a)}{\Rightarrow} \quad \frac{n^2 + 6 + 3(x-2)}{6} = 6uv - u + v \quad \Rightarrow$$
$$\Rightarrow \quad \frac{n^2 + 6 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 6u}{6(6u-1)} - u + \frac{n^2 + 3(x-1) + 6u}{6(6u-1)} =$$

(15b)
$$= \frac{6un^2 + 18ux - 6u + n^2 + 3x - 3}{6(6u-1)} \quad \Rightarrow$$
$$\Rightarrow \quad \left(n^2 + 6 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux - 6u + n^2 + 3x - 3 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - n^2 - 3x = 6un^2 + 18ux - 6u + n^2 + 3x - 3 \quad \Rightarrow$$
$$\Rightarrow \quad 6u = 2n^2 + 6x - 3 \quad \Rightarrow \quad u = \frac{2n^2 + 6x - 3}{6} = x + \underbrace{\frac{n^2}{3} - \frac{1}{2}}_{never\ an\ \text{int}eger}$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (15a)-(15b) is contrary!

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Case of the (3b),(3c) - (10c) – (10a):

(16a)
$$g_x = g_1 + 3(x-1) \overset{(3b),(3c)}{=} n^2 + 1 + 3(x-1) \overset{(10c)}{=} 6(6uv + u - v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3b),(3c)}{=} n^2 + 3 + 3(x-2) \overset{(10a)}{=} 6(6uv + u + v) + 1$$

$$\overset{(16a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 2}{6} = v(6u-1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} \quad \overset{(16a)}{\Rightarrow}$$
$$\overset{(16a)}{\Rightarrow} \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6uv + u + v \quad \Rightarrow$$
(16b)
$$\Rightarrow \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} + u + \frac{n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 12u - 36u^2 + 36u^2 - 6u + n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18ux - 12u + 3x + n^2 - 1}{6(6u-1)}$$

$$\overset{(16b)}{\Rightarrow} \quad \left(n^2 + 2 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux - 12u + 3x + n^2 - 1 \quad \Rightarrow$$
(16c)
$$\Rightarrow \quad 6un^2 + 18ux - 24u - n^2 - 3x + 4 = 6un^2 + 18ux - 12u + 3x + n^2 - 1 \quad \Rightarrow$$
$$\Rightarrow \quad 2n^2 + 12u + 3x - 3 = 0 \quad \Rightarrow \quad u = \frac{3 - 2n^2 - 3x}{12} \langle\ 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (16a)-(16c) is contrary!

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Case of the (3b),(3c) - (10c) – (10b):

(17a)
$$g_x = g_1 + 3(x-1) \overset{(3b),(3c)}{=} n^2 + 1 + 3(x-1) \overset{(10c)}{=} 6(6uv + u - v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3b),(3c)}{=} n^2 + 3 + 3(x-2) \overset{(10b)}{=} 6(6uv - u - v) + 1$$

$$\overset{(17a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 2}{6} = v(6u-1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} \quad \overset{(17a)}{\Rightarrow}$$
$$\overset{(17a)}{\Rightarrow} \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6uv - u - v \quad \Rightarrow$$

(17b)
$$\Rightarrow \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} - u - \frac{n^2 + 3(x-1) + 2 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 12u - 36u^2 - 36u^2 + 6u - n^2 - 3(x-1) - 2 + 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18ux + 6u - 72u^2 - 3x - n^2 + 1}{6(6u-1)}$$

(17c)
$$\overset{(17b)}{\Rightarrow} \quad \left(n^2 + 2 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux + 6u - 72u^2 - 3x - n^2 + 1 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 24u - n^2 - 3x + 4 = 6un^2 + 18ux + 6u - 72u^2 - 3x - n^2 + 1 \quad \Rightarrow$$
$$\Rightarrow \quad 72u^2 - 30u - 1 = 0 \quad \Rightarrow \quad u_{1,2} = \frac{30 \pm \sqrt{30^2 + 4 \cdot 72}}{2 \cdot 72} = \underbrace{\frac{15 \pm \sqrt{15^2 + 72}}{72}}_{not\ an\ \text{int}eger}$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (17a)-(17c) is contrary!

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Case of the (3b),(3c) - (10d) – (10a):

(18a)
$$g_x = g_1 + 3(x-1) \overset{(3b),(3c)}{=} n^2 + 1 + 3(x-1) \overset{(10d)}{=} 6(6uv - u + v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3b),(3c)}{=} n^2 + 3 + 3(x-2) \overset{(10a)}{=} 6(6uv + u + v) + 1$$

(18b)
$$\overset{(18a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 2}{6} = v(6u+1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} \quad \overset{(18a)}{\Rightarrow}$$
$$\overset{(18a)}{\Rightarrow} \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6uv + u + v \quad \Rightarrow$$
$$\Rightarrow \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} + u + \frac{n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 12u + 36u^2 + 36u^2 + 6u + n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18ux + 6u + 72u^2 + 3x + n^2 - 1}{6(6u+1)}$$

(18c)
$$\overset{(18b)}{\Rightarrow} \quad \left(n^2 + 2 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux + 6u + 72u^2 + 3x + n^2 - 1 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 24u + n^2 + 3x - 4 = 6un^2 + 18ux + 6u + 72u^2 + 3x + n^2 - 1 \quad \Rightarrow$$
$$\Rightarrow \quad 24u^2 + 10u + 1 = 0 \quad \Rightarrow \quad u_{1,2} = \frac{-10 \pm \sqrt{100 - 96}}{48} = \frac{-10 \pm 2}{48} \langle \ 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (18a)-(18c) is contrary!

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Case of the (3b),(3c) - (10d) – (10b):

(19a)
$$g_x = g_1 + 3(x-1) \overset{(3b),(3c)}{=} n^2 + 1 + 3(x-1) \overset{(10d)}{=} 6(6uv - u + v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3b),(3c)}{=} n^2 + 3 + 3(x-2) \overset{(10b)}{=} 6(6uv - u - v) + 1$$

$$\overset{(19a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 2}{6} = v(6u+1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} \quad \overset{(19a)}{\Rightarrow}$$
$$\overset{(19a)}{\Rightarrow} \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6uv - u - v \quad \Rightarrow$$

(19b)
$$\Rightarrow \quad \frac{n^2 + 2 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} - u - \frac{n^2 + 3(x-1) + 2 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 12u + 36u^2 - 36u^2 - 6u - n^2 - 3(x-1) - 2 - 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18ux - 18u - 3x - n^2 + 1}{6(6u+1)}$$

(19c)
$$\overset{(19b)}{\Rightarrow} \quad \left(n^2 + 2 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux - 18u - 3x - n^2 + 1 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 24u + n^2 + 3x - 4 = 6un^2 + 18ux + 6u + 72u^2 + 3x + n^2 - 1 \quad \Rightarrow$$
$$\Rightarrow \quad 2n^2 + 6x - 5 = 6u \quad \Rightarrow \quad u = \frac{2n^2 + 6x - 5}{6} = \underbrace{\frac{2n^2 - 5}{6}}_{never\ an\ \text{int}eger} + x$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (19a)-(19c) is contrary!

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Case of the (3d),(3e) - (10c) – (10a):

(20a)
$$g_x = g_1 + 3(x-1) \overset{(3d),(3e)}{=} n^2 + 4 + 3(x-1) \overset{(10c)}{=} 6(6uv + u - v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3d),(3e)}{=} n^2 + 6 + 3(x-2) \overset{(10a)}{=} 6(6uv + u + v) + 1$$

$$\overset{(20a)}{\Rightarrow} \frac{n^2 + 3(x-1) + 5}{6} = v(6u-1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} \quad \overset{(20a)}{\Rightarrow}$$
$$\overset{(20a)}{\Rightarrow} \frac{n^2 + 5 + 3(x-2)}{6} = 6uv + u + v \quad \Rightarrow$$

(20b)
$$\Rightarrow \frac{n^2 + 5 + 3(x-2)}{6} = 6u \frac{n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} + u + \frac{n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 30u - 36u^2 + 36u^2 - 6u + n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18ux + 3x + n^2 + 2}{6(6u-1)}$$

(20c)
$$\overset{(20b)}{\Rightarrow} \left(n^2 + 5 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux + 3x + n^2 + 2 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 6u - n^2 - 3x + 1 = 6un^2 + 18ux + 3x + n^2 + 2 \quad \Rightarrow$$
$$\Rightarrow \quad -2n^2 - 6x - 1 = 6u \quad \Rightarrow \quad u = \frac{-2n^2 - 6x - 1}{6} \langle \, 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (20a)-(20c) is contrary!

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Case of the (3d),(3e) - (10c) – (10b):

(21a)
$$g_x = g_1 + 3(x-1) \overset{(3d),(3e)}{=} n^2 + 4 + 3(x-1) \overset{(10c)}{=} 6(6uv + u - v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3d),(3e)}{=} n^2 + 6 + 3(x-2) \overset{(10b)}{=} 6(6uv - u - v) + 1$$

$$\overset{(21a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 5}{6} = v(6u-1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} \quad \overset{(21a)}{\Rightarrow}$$
$$\overset{(21a)}{\Rightarrow} \quad \frac{n^2 + 5 + 3(x-2)}{6} = 6uv - u - v \quad \Rightarrow$$

(21b)
$$\Rightarrow \quad \frac{n^2 + 5 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} - u - \frac{n^2 + 3(x-1) + 5 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 30u - 36u^2 - 36u^2 + 6u - n^2 - 3(x-1) - 5 + 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18ux - 72u^2 + 24u - 3x - n^2 - 2}{6(6u-1)}$$

(21c)
$$\overset{(21b)}{\Rightarrow} \quad \left(n^2 + 5 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux - 72u^2 + 24u - 3x - n^2 - 2 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 6u - n^2 - 3x + 1 = 6un^2 + 18ux - 72u^2 + 24u - 3x - n^2 - 2 \quad \Rightarrow$$
$$\Rightarrow \quad 72u^2 - 30u = 0 \quad \Rightarrow \quad u(72u - 30) = 0 \quad \Rightarrow \quad u_1 = 0, \quad u_2 = \frac{30}{72}$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (21a)-(21c) is contrary!

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Case of the (3d),(3e) - (10d) – (10a):

(22a)
$$g_x = g_1 + 3(x-1) \overset{(3d),(3e)}{=} n^2 + 4 + 3(x-1) \overset{(10d)}{=} 6(6uv - u + v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3d),(3e)}{=} n^2 + 6 + 3(x-2) \overset{(10a)}{=} 6(6uv + u + v) + 1$$

$$\overset{(22a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 5}{6} = v(6u+1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 5 + 6u}{6(6u+1)} \quad \overset{(22a)}{\Rightarrow}$$
$$\overset{(22a)}{\Rightarrow} \quad \frac{n^2 + 5 + 3(x-2)}{6} = 6uv + u + v \quad \Rightarrow$$

(22b)
$$\Rightarrow \quad \frac{n^2 + 5 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 5 + 6u}{6(6u+1)} + u + \frac{n^2 + 3(x-1) + 5 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 30u + 36u^2 + 36u^2 + 6u + n^2 + 3x + 2 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18ux - 72u^2 + 24u + 3x + n^2 + 2}{6(6u+1)}$$

(22c)
$$\overset{(22b)}{\Rightarrow} \quad \left(n^2 + 5 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux - 72u^2 + 24u + 3x + n^2 + 2 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 6u + n^2 + 3x - 1 = 6un^2 + 18ux - 72u^2 + 24u - 3x - n^2 - 2 \quad \Rightarrow$$
$$\Rightarrow \quad 72u^2 - 24u - 3 = 0 \quad \Rightarrow \quad 24u^2 - 8u - 1 = 0 \quad \Rightarrow \quad u_{1,2} = \underbrace{\frac{2 \pm \sqrt{10}}{12}}_{not\ an\ \mathrm{int}\, eger}$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (22a)-(22c) is contrary!

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Case of the (3d),(3e) - (10d) – (10b):

(23a)
$$g_x = g_1 + 3(x-1) \overset{(3d),(3e)}{=} n^2 + 4 + 3(x-1) \overset{(10d)}{=} 6(6uv - u + v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3d),(3e)}{=} n^2 + 6 + 3(x-2) \overset{(10b)}{=} 6(6uv - u - v) + 1$$

$$\overset{(23a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 5}{6} = v(6u+1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 5 + 6u}{6(6u+1)} \quad \overset{(23a)}{\Rightarrow}$$
$$\overset{(23a)}{\Rightarrow} \quad \frac{n^2 + 5 + 3(x-2)}{6} = 6uv - u - v \quad \Rightarrow$$

(23b)
$$\Rightarrow \quad \frac{n^2 + 5 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 5 + 6u}{6(6u+1)} - u - \frac{n^2 + 3(x-1) + 5 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 30u + 36u^2 - 36u^2 - 6u - n^2 - 3x - 2 - 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18ux - 3x - n^2 - 2}{6(6u+1)}$$

$$\overset{(23b)}{\Rightarrow} \quad \left(n^2 + 5 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux - 3x - n^2 - 2 \quad \Rightarrow$$

(23c)
$$\Rightarrow \quad 6un^2 + 18ux - 6u + n^2 + 3x - 1 = 6un^2 + 18ux - 3x - n^2 - 2 \quad \Rightarrow$$
$$\Rightarrow \quad -6u + 2n^2 + 6x + 1 = 0 \quad \Rightarrow \quad u = \frac{2n^2 + 6x + 1}{6} = x + \underbrace{\frac{2n^2 + 1}{6}}_{never\ an\ \mathrm{int}eger}$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (23a)-(23c) is contrary!

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Case of the (3f) - (10c) – (10a):

(24a)
$$g_x = g_1 + 3(x-1) \overset{(3f)}{=} n^2 + 2 + 3(x-1) \overset{(10c)}{=} 6(6uv + u - v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3f)}{=} n^2 + 4 + 3(x-2) \overset{(10a)}{=} 6(6uv + u + v) + 1$$

$$\overset{(24a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 3}{6} = v(6u-1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 3 - 6u}{6(6u-1)} \quad \overset{(24a)}{\Rightarrow}$$
$$\overset{(24a)}{\Rightarrow} \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6uv + u + v \quad \Rightarrow$$

(24b)
$$\Rightarrow \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 3 - 6u}{6(6u-1)} + u + \frac{n^2 + 3(x-1) + 3 - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 18u - 36u^2 + 36u^2 - 6u + n^2 + 3x - 6u}{6(6u-1)} =$$
$$= \frac{6un^2 + 18ux - 12u + 3x + n^2}{6(6u-1)}$$

(24c)
$$\overset{(24b)}{\Rightarrow} \quad \left(n^2 + 3 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux - 12u + 3x + n^2 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 18u - n^2 - 3x + 3 = 6un^2 + 18ux - 12u + 3x + n^2 \quad \Rightarrow$$
$$\Rightarrow \quad 6u + 2n^2 + 6x + 6 = 0 \quad \Rightarrow \quad u = -\frac{2n^2 + 6x + 6}{6} \langle \ 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (24a)-(24c) is contrary!

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Case of the (3f) - (10c) – (10b):

(25a)
$$g_x = g_1 + 3(x-1) \overset{(3f)}{=} n^2 + 2 + 3(x-1) \overset{(10c)}{=} 6(6uv + u - v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3f)}{=} n^2 + 4 + 3(x-2) \overset{(10b)}{=} 6(6uv - u - v) + 1$$

$$\overset{(25a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 3}{6} = v(6u-1) + u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 3 - 6u}{6(6u-1)} \quad \overset{(25a)}{\Rightarrow}$$

$$\overset{(25a)}{\Rightarrow} \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6uv - u - v \quad \Rightarrow$$

(25b)
$$\Rightarrow \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 3 - 6u}{6(6u-1)} - u - \frac{n^2 + 3(x-1) + 3 - 6u}{6(6u-1)} =$$

$$= \frac{6un^2 + 18u(x-1) + 18u - 36u^2 - 36u^2 + 6u - n^2 - 3x + 6u}{6(6u-1)} =$$

$$= \frac{6un^2 + 18ux - 72u^2 + 12u - 3x - n^2}{6(6u-1)}$$

(25c)
$$\overset{(25b)}{\Rightarrow} \quad \left(n^2 + 3 + 3(x-2)\right)(6u-1) = 6un^2 + 18ux - 72u^2 + 12u - 3x - n^2 \quad \Rightarrow$$

$$\Rightarrow \quad 6un^2 + 18ux - 18u - n^2 - 3x + 3 = 6un^2 + 18ux - 72u^2 + 12u - 3x - n^2 \quad \Rightarrow$$

$$\Rightarrow \quad 72u^2 - 30u - 3 = 0 \quad \Rightarrow \quad 24u^2 - 10u - 1 = 0 \quad \Rightarrow \quad u_{1,2} = \frac{10 \pm 14}{48} \langle 1$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (25a)-(25c) is contrary!

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Case of the (3f) - (10d) – (10a):

(26a)
$$g_x = g_1 + 3(x-1) \overset{(3f)}{=} n^2 + 2 + 3(x-1) \overset{(10d)}{=} 6(6uv - u + v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3f)}{=} n^2 + 4 + 3(x-2) \overset{(10a)}{=} 6(6uv + u + v) + 1$$

$$\overset{(26a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 3}{6} = v(6u+1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 3 + 6u}{6(6u+1)} \quad \overset{(26a)}{\Rightarrow}$$
$$\overset{(26a)}{\Rightarrow} \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6uv + u + v \quad \Rightarrow$$

(26b)
$$\Rightarrow \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 3 + 6u}{6(6u+1)} + u + \frac{n^2 + 3(x-1) + 3 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 18u + 36u^2 + 36u^2 + 6u + n^2 + 3x + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18ux + 72u^2 + 12u + 3x + n^2}{6(6u+1)}$$

$$\overset{(26b)}{\Rightarrow} \quad \left(n^2 + 3 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux - 18u + 3x + n^2 - 3 \quad \Rightarrow$$

(26c)
$$\Rightarrow \quad 6un^2 + 18ux - 18u + n^2 + 3x - 3 = 6un^2 + 18ux + 72u^2 + 12u + 3x + n^2 \quad \Rightarrow$$
$$\Rightarrow \quad 72u^2 + 30u + 3 = 0 \quad \Rightarrow \quad 24u^2 + 10u + 1 = 0 \quad \Rightarrow \quad u_{1,2} = \frac{-10 \pm 2}{48} \langle \ 0$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (26a)-(26c) is contrary!

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Case of the (3f) - (10d) – (10b):

(27a)
$$g_x = g_1 + 3(x-1) \overset{(3f)}{=} n^2 + 2 + 3(x-1) \overset{(10d)}{=} 6(6uv - u + v) - 1$$
$$g_{x+1} = g_2 + 3(x-2) \overset{(3f)}{=} n^2 + 4 + 3(x-2) \overset{(10b)}{=} 6(6uv - u - v) + 1$$

$$\overset{(27a)}{\Rightarrow} \quad \frac{n^2 + 3(x-1) + 3}{6} = v(6u+1) - u \Rightarrow \quad v = \frac{n^2 + 3(x-1) + 3 + 6u}{6(6u+1)} \quad \overset{(27a)}{\Rightarrow}$$

$$\overset{(27a)}{\Rightarrow} \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6uv - u - v \quad \Rightarrow$$

(27b)
$$\Rightarrow \quad \frac{n^2 + 3 + 3(x-2)}{6} = 6u\frac{n^2 + 3(x-1) + 3 + 6u}{6(6u+1)} - u - \frac{n^2 + 3(x-1) + 3 + 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18u(x-1) + 18u + 36u^2 - 36u^2 - 6u - n^2 - 3x - 6u}{6(6u+1)} =$$
$$= \frac{6un^2 + 18ux - 12u - 3x - n^2}{6(6u+1)}$$

(27c)
$$\overset{(27b)}{\Rightarrow} \quad \left(n^2 + 3 + 3(x-2)\right)(6u+1) = 6un^2 + 18ux - 18u + 3x + n^2 - 3 \quad \Rightarrow$$
$$\Rightarrow \quad 6un^2 + 18ux - 18u + n^2 + 3x - 3 = 6un^2 + 18ux - 12u - 3x - n^2 \quad \Rightarrow$$
$$\Rightarrow \quad 6u - 2n^2 - 6x + 3 = 0 \quad \Rightarrow \quad u = x + \underbrace{\frac{2n^2 - 3}{6}}_{never\ an\ \text{int}eger}$$

u is defined as a natural number (see (10a)-(10d)), but the result of calculations (27a)-(27c) is contrary!

We have proved above that there are always *t* pieces of $g_1,...,g_t$ *6z±1*-type natural numbers in the interval $\left[n^2,(n+1)^2\right]$ (see (9a) - (9f)) and only among these can be the prime numbers in this interval.

However, in the above (12a)-(27c) has been proved that the indirect condition for any *n* natural number is never satisfied. It follows that, for any natural number *n*, there exists a prime number among the (5) and (7) type $g_1,\dots,g_t$ natural numbers. This is exactly the proof of item (1). This is exactly proof of (1) statement of our Theorem 1.

Q.E.D.

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Table 1.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
n	n^2	$(n+1)^2$		type	g_1		type	g_2	t
3	9	16	(3f)$\Rightarrow g_1=n^2+2$	6z-1	11	(4f)$\Rightarrow g_2=n^2+4$	6z+1	13	2
4	16	25	(3b)$\Rightarrow g_1=n^2+1$	6z-1	17	(4b)$\Rightarrow g_2=n^2+3$	6z+1	19	3
5	25	36	(3e)$\Rightarrow g_1=n^2+4$	6z-1	29	(4e)$\Rightarrow g_2=n^2+6$	6z+1	31	3
6	36	49	(3a)$\Rightarrow g_1=n^2+1$	6z+1	37	(4a)$\Rightarrow g_2=n^2+5$	6z-1	41	4
7	49	64	(3d)$\Rightarrow g_1=n^2+4$	6z-1	53	(4d)$\Rightarrow g_2=n^2+6$	6z+1	55	4
8	64	81	(3c)$\Rightarrow g_1=n^2+1$	6z-1	65	(4c)$\Rightarrow g_2=n^2+3$	6z+1	67	6
9	81	100	(3f)$\Rightarrow g_1=n^2+2$	6z-1	83	(4f)$\Rightarrow g_2=n^2+4$	6z+1	85	6
10	100	121	(3b)$\Rightarrow g_1=n^2+1$	6z-1	101	(4b)$\Rightarrow g_2=n^2+3$	6z+1	103	7
11	121	144	(3e)$\Rightarrow g_1=n^2+4$	6z-1	125	(4e)$\Rightarrow g_2=n^2+6$	6z+1	127	7
12	144	169	(3a)$\Rightarrow g_1=n^2+1$	6z+1	145	(4a)$\Rightarrow g_2=n^2+5$	6z-1	149	8
13	169	196	(3d)$\Rightarrow g_1=n^2+4$	6z-1	173	(4d)$\Rightarrow g_2=n^2+6$	6z+1	175	8
14	196	225	(3c)$\Rightarrow g_1=n^2+1$	6z-1	197	(4c)$\Rightarrow g_2=n^2+3$	6z+1	199	10
…									
51	2601	2704	(3f)$\Rightarrow g_1=n^2+2$	6z-1	2603	(4f)$\Rightarrow g_2=n^2+4$	6z+1	2605	34
			$g_3=g_1+6$	6z-1	2609				
52	2704	2809	(3b)$\Rightarrow g_1=n^2+1$	6z-1	2705	(4b)$\Rightarrow g_2=n^2+3$	6z+1	2707	35
53	2809	2916	(3e)$\Rightarrow g_1=n^2+4$	6z-1	2813	(4e)$\Rightarrow g_2=n^2+6$	6z+1	2815	35
			$g_3=g_1+6$	6z-1	2819				
54	2916	3025	(3a)$\Rightarrow g_1=n^2+1$	6z+1	2917	(4a)$\Rightarrow g_2=n^2+5$	6z-1	2921	36
55	3025	3136	(3d)$\Rightarrow g_1=n^2+4$	6z-1	3029	(4d)$\Rightarrow g_2=n^2+6$	6z+1	3031	36
			$g_3=g_1+6$	6z-1	3035	$g_4=g_2+6$	6z+1	3037	
56	3136	3249	(3c)$\Rightarrow g_1=n^2+1$	6z-1	3137	(4c)$\Rightarrow g_2=n^2+3$	6z+1	3139	38
…									

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Table 2.

n	n^2	$(n+1)^2$	$n^2 \langle q \langle (n+1)^2$
1	*1*	*4*	***2***
2	*4*	*9*	***5***
3	*9*	*16*	***11***
4	*16*	*25*	***19***
5	*25*	*36*	***31***
6	*36*	*49*	***41***
7	*49*	*64*	***61***
8	*64*	*81*	***67***
9	*81*	*100*	***83***
10	*100*	*121*	***103***
11	*121*	*144*	***139***
12	*144*	*169*	***149***
13	*169*	*196*	***181***
…			
100	*10.000*	*10.201*	***10.069***
…			
1.000	*1.000.000*	*1.002.001*	***1.000.039***
…			
10.000	*100.000.000*	*100.020.001*	***100.000.213***
…			

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