

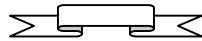
Properties of twin prime-based Mersenne twin primes

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Abstract

In the present paper, we show that there are no two adjacent Mersenne primes that would be twin primes. Then we define the *Mersenne twin primes* based on the twin-prime pair and examine their basic properties.



By the Theorem 2 in [Dénes 2001c], if $p > 3$ prime number and $M_p = 2^p - 1$ Mersenne prime, then the two cases (1), (2) are true:

$$(1) \quad p = 6k - 1 \quad (k = 1, 2, 3, \dots) \Rightarrow M_p = \left(6 \sum_{i=0}^{3k-2} 4^i \right) + 1$$

$$(2) \quad p = 6k + 1 \quad (k = 1, 2, 3, \dots) \Rightarrow M_p = \left(6 \sum_{i=0}^{3k-1} 4^i \right) + 1$$

We employ the known mathematical relationship (3).

$$(3) \quad a^n - 1 = (a - 1)(a^{n-1} + a^{n-2} + a^{n-3} + \dots + a^1 + a^0) = (a - 1) \sum_{i=0}^{n-1} a^i \Rightarrow \sum_{i=0}^{n-1} a^i = \frac{a^n - 1}{a - 1}$$

and introduce notations (4), (5).

$$(4) \quad m^-(k) = \sum_{i=0}^{3k-2} 4^i = \frac{4^{3k-1} - 1}{3} \quad (k = 1, 2, 3, \dots)$$

$$(5) \quad m^+(k) = \sum_{i=0}^{3k-1} 4^i = \frac{4^{3k} - 1}{3} \quad (k = 1, 2, 3, \dots)$$

By the Theorem 1 in [Dénes 2001c], if $M_p = 2^p - 1$ is a Mersenne prime, then it is always $6K + 1$ form (K is a natural number).

On the other hand, we know about twin primes that if p and $p + 2$ are twin primes, then $p = 6k - 1$ and $p + 2 = 6k + 1$ form (k is a natural number). So two Mersenne primes can never be twin primes.

However, we can examine the successive M_p, M_{p+2} Mersenne numbers assigned to the twin primes $p, p + 2$.

DEFINITION 1. (*Mersenne twin prime*)

Let M_p, M_{p+2} be Mersenne numbers assigned to the twin primes $p > 3, p + 2$.

If $M_p = 2^p - 1$ and $M_{p+2} = 2^{p+2} - 1$ are prime numbers, then we call them *Mersenne twin prime assigned to the twin primes $p, p + 2$* .

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It follows from definition 1. for the *Mersenne twin prime assigned to the twin primes p, p+2*:

$$(6) \quad p = 6k - 1, p + 2 = 6k + 1 (k = 1,2,3,\dots) \Rightarrow M_p = 2^{6k-1} - 1, M_{p+2} = 2^{6k+1} - 1$$

$$(7) \quad M_{p+2} - M_p = 2^{6k+1} - 2^{6k-1} = 3 \cdot 2^{6k-1} = 3(M_p + 1) \Rightarrow M_{p+2} = 4M_p + 3$$

The result (7) for Mersenne twin primes is demonstrated 1.-2., 5.-6. and 19.-20. rows of Table 1.

Of course, the relation (7) derived for successive Mersenne numbers is valid in all cases if $k^+ = k^- = k$ (using the notations in Table 1.), thus for the Mersenne number pair M_p and M_{p+2} , the following four cases are theoretically possible (see (8)):

(8)

	a. <i>Mersenne-twin primes</i>	b.	c.	d.
M_p	<i>prime</i>	<i>prime</i>	<i>Not prime</i>	<i>Not prime</i>
M_{p+2}	<i>prime</i>	<i>Not prime</i>	<i>prime</i>	<i>Not prime</i>
<i>Example in the rows of Table 1</i>	1.-2. 5.-6. 19.-20.	NOT POSSIBLE! <i>(See Theorem 1)</i>	3.-4. 9.-10.	13.-14.

An interesting problem arises that the (8)/b column case, we do not find an example among the first twenty Mersenne numbers (see Table 1). This is due to the following Theorem 1.

THEOREM 1.

If M_p is a Mersenne-prime, then M_{p+2} cannot be a composite number.

PROOF

On the basis of (7) connection $M_{p+2} = 4M_p + 3$, which can be a composite number if and only if M_p is divisible by 3. This, in turn contradicts the fact that the condition of the theorem is that M_p is Mersenne prime.

Q.E.D.

From the Theorem 1. following the next Theorem 2.

THEOREM 2.

If $p, p+2$ are twin pimes and M_p Mersenne-prime, then M_{p+2} is Mersenne-prime too.

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OPEN PROBLEM:

Are there infinitely many $p, p+2$ twin primes for which there is $p, p+2$ based M_p, M_{p+2} Mersenne twin prime?

Assume that there are a finite number of $p, p+2$ based M_p, M_{p+2} Mersenne twin prime. Then there exists a prime number q such that for all $p=6k-1$ form prime number ($p>q$) then M_p, M_{p+2} are not Mersenne twin primes. According to connections (8), this can only be if M_p is not a prime number.

The answer to the above open problem is therefore equivalent to proving or refuting the following Theorem 3.

THEOREM 3.

Véges sok $p=6k-1$ (k term.szám) prímszám létezik, amelyre $M_p=2^p-1$ Mersenne-prím.

There are a finite number of $p=6k-1$ form prime number (k natural number) on which $M_p=2^p-1$ is Mersenne-prime.

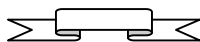


Table 1.

	k^-	k^+	$p = 6k \pm 1$	Mersenne-numbers (M_p)
1.	1		5	$M_5=2^5-1=31$ (prime)
2.		1	7	$M_7=2^7-1=127$ (prime)
3.	2		11	$M_{11}=2^{11}-1=2.047=(6 \cdot 4-1)(6 \cdot 15-1)$
4.		2	13	$M_{13}=2^{13}-1=8.191$ (prime)
5.	3		17	$M_{17}=2^{17}-1=131.071$ (prime)
6.		3	19	$M_{19}=2^{19}-1=524.287$ (prime)
7.	4		23	$M_{23}=2^{23}-1=8.388.607=(6 \cdot 8-1)(6 \cdot 29.747-1)$
8.		4	25	Not Mersenne-number $2^{25}-1=33.554.431=(6 \cdot 5+1)(6 \cdot 100+1)(6 \cdot 300+1)$
9.	5		29	$M_{29}=2^{29}-1=536.870.911=(6 \cdot 39-1)(6 \cdot 384.028-1)$
10.		5	31	$M_{31}=2^{31}-1=2.147.483.647$ (prime)
11.	6		35	Not Mersenne-number $2^{35}-1=34.359.738.367=(6 \cdot 5+1)(6 \cdot 12-1)(6 \cdot 21+1)(6 \cdot 20.487-1)$
12.		6	37	$M_{37}=2^{37}-1=137.438.953.471=(6 \cdot 37+1)(6 \cdot 102.719.696+1)$
13.	7		41	$M_{41}=2^{41}-1=2.199.023.255.551=(6 \cdot 2.228-1)(6 \cdot 27.418.559-1)$
14.		7	43	$M_{43}=2^{43}-1=8.796.093.022.207=(6 \cdot 698.148+1)(6 \cdot 349.977+1)$
15.	8		47	$M_{47}=2^{47}-1=140.737.488.355.327=(6 \cdot 392-1)(6 \cdot 9.977.136.563-1)$
16.		8	49	Not Mersenne-number $2^{49}-1=562.949.953.421.311=(6 \cdot 21+1)(6 \cdot 738.779.466.432+1)$
17.	9		53	$M_{53}=2^{53}-1=9.007.199.254.740.991=(6 \cdot 11.572-1)(6 \cdot 21.621.464.127-1)$
18.		9	55	Not Mersenne-number $2^{55}-1=36.028.797.018.963.967=(6 \cdot 4-1)(6 \cdot 5+1)(6 \cdot 15-1)(6 \cdot 147-1)(6 \cdot 532-1)(6 \cdot 33.660+1)$
19.	10		59	$M_{59}=2^{59}-1=576.460.752.303.423.487$ (prime)
20.		10	61	$M_{61}=2^{61}-1=2.305.843.009.213.693.951$ (prime)
21.	11		65	Not Mersenne-number $2^{65}-1=36.893.488.147.419.103.231=(6 \cdot 5+1)(6 \cdot 1.365+1)(6 \cdot 24.215.857.259.685+1)$
22.		11	67	$M_{67}=2^{67}-1=147.573.952.589.676.412.927=(6 \cdot 32.284.620+1)(6 \cdot 126.973.042.881+1)$

References

[Dénes 2001a] Complementary prime-sieve P_Ure Mathematics and Applications, Vol.12 (2001), No. 2, pp. 197-207

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[Dénes 2001b] Komplementer prímszita és alkalmazása a prímszámok számának becslésére

http://www.titoktan.hu/_raktar/_e_vilagi_gondolatok/KomplementerPrimszita.pdf

[Dénes 2001c] Basic properties of Mersenne-numbers

(Parallel algorithm for prime factorization of Mersenne-numbers)

http://www.titoktan.hu/_raktar/_e_vilagi_gondolatok/Mersenne-primes1.pdf