Properties of twin prime-based Mersenne twin primes

Dénes, Tamás mathematician

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Abstract
In the present paper, we show that there are no two adjacent Mersenne primes that would be twin primes. Then we define the *Mersenne twin primes* based on the twin-prime pair and examine their basic properties.

By the Theorem 2 in [Dénes 2001c], if $p>3$ prime number and $M_p=2^p-1$ Mersenne prime, then the two cases (1), (2) are true:

(1) $p=6k-1$ ($k=1,2,3,\ldots$) $\Rightarrow M_p=\left(6\sum_{i=0}^{3k-2}4^i\right)+1$

(2) $p=6k+1$ ($k=1,2,3,\ldots$) $\Rightarrow M_p=\left(6\sum_{i=0}^{3k-1}4^i\right)+1$

We employ the known mathematical relationship (3).

(3) $a^n-1=(a-1)(a^{n-1}+a^{n-2}+a^{n-3}+\ldots+a^1+a^0)=(a-1)\sum_{i=0}^{n-1}a^i \Rightarrow \sum_{i=0}^{n-1}a^i=\frac{a^n-1}{a-1}$

and introduce notations (4), (5).

(4) $m^-(k)=\sum_{i=0}^{3k-2}4^i\overset{(3)}{=}\frac{4^{3k-1}-1}{3}$ ($k=1,2,3,\ldots$)

(5) $m^+(k)=\sum_{i=0}^{3k-1}4^i\overset{(3)}{=}\frac{4^{3k}-1}{3}$ ($k=1,2,3,\ldots$)

By the Theorem 1 in [Dénes 2001c], if $M_p=2^p-1$ is a Mersenne prime, then it is always $6K+1$ form (K is a natural number).
On the other hand, we know about twin primes that if p and $p+2$ are twin primes, then $p=6k-1$ and $p+2=6k+1$ form (k is a natural number). So two Mersenne primes can never be twin primes.
However, we can examine the successive M_p, M_{p+2} Mersenne numbers assigned to the twin primes p, $p+2$.

DEFINITION 1. (*Mersenne twin prime*)
Let M_p, M_{p+2} be Mersenne numbers assigned to the twin primes $p>3$, $p+2$.
If $M_p=2^p-1$ and $M_{p+2}=2^{p+2}-1$ are prime numbers, then we call them *Mersenne twin prime assigned to the twin primes p, p+2.*

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It follows from definition 1. for the *Mersenne twin prime assigned to the twin primes p, p+2*:

(6) $$p = 6k - 1,\ p + 2 = 6k + 1\ (k = 1,2,3,...) \ \Rightarrow\ M_p = 2^{6k-1} - 1,\quad M_{p+2} = 2^{6k+1} - 1$$

(7) $$M_{p+2} - M_p = 2^{6k+1} - 2^{6k-1} = 3 \cdot 2^{6k-1} = 3(M_p + 1) \ \Rightarrow\ M_{p+2} = 4M_p + 3$$

The result (7) for Mersenne twin primes is demonstrated 1.–2., 5.–6. and 19.-20. rows of Table 1.
Of course, the relation (7) derived for successive Mersenne numbers is valid in all cases if $k^{-}=k^{+}=k$ (using the notations in Table 1.), thus for the Mersenne number pair M_p and M_{p+2} , the following four cases are theoretically possible (see (8)):

(8)

	a. *Mersenne-twin primes*	**b.**	**c.**	**d.**
M_p	***prime***	***prime***	***Not prime***	***Not prime***
M_{p+2}	***prime***	***Not prime***	***prime***	***Not prime***
Example in the rows of Table 1	1.-2. 5.-6. 19.-20.	*NOT POSSIBLE! (See Theorem 1)*	3.-4. 9.-10.	13.-14.

An interesting problem arises that the (8)/b column case, we do not find an example among the first twenty Mersenne numbers (see Table 1). This is due to the following Theorem 1.

THEOREM 1.
If M_p is a Mersenne-prime, then M_{p+2} cannot be a compozite number.

PROOF
On the basis of (7) connection $M_{p+2}=4M_p+3$, which can be a compozite number if and only if M_p is divisible by 3. This, in turn contradicts the fact that the condition of the theorem is that M_p is Mersenne prime.

Q.E.D.

From the Theorem 1. following the next Theorem 2.

THEOREM 2.
If p, $p+2$ are twin pimes and M_p Mersenne-prime, then M_{p+2} is Mersenne-prime too.

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OPEN PROBLEM:
Are there infinitely many p, $p+2$ twin primes for which there is p, $p+2$ based M_p, M_{p+2} Mersenne twin prime?

Assume that there are a finite number of p, $p+2$ based M_p, M_{p+2} Mersenne twin prime. Then there exists a prime number q such that for all $p=6k-1$ form prime number ($p>q$) then M_p, M_{p+2} are not Mersenne twin primes. According to connections (8), this can only be if M_p is not a prime number.

The answer to the above open problem is therefore equivalent to proving or refuting the following Theorem 3.

THEOREM 3.
Véges sok $p=6k-1$ (k term.szám) prímszám létezik, amelyre $M_p=2^p-1$ Mersenne-prím.
There are a finite number of $p=6k-1$ form prime number (k natural number) on which $M_p=2^p-1$ is Mersenne-prime.

Table 1.

	k^-	k^+	$p = 6k \pm 1$	Mersenne-numbers (M_p)
1.	1		5	$M_5=2^5-1=$**31 (prime)**
2.		1	7	$M_7=2^7-1=$**127 (prime)**
3.	2		11	$M_{11}=2^{11}-1=$**2.047**$=(6\cdot4-1)(6\cdot15-1)$
4.		2	13	$M_{13}=2^{13}-1=$**8.191 (prime)**
5.	3		17	$M_{17}=2^{17}-1=$**131.071 (prime)**
6.		3	19	$M_{19}=2^{19}-1=$**524.287 (prime)**
7.	4		23	$M_{23}=2^{23}-1=$**8.388.607**$=(6\cdot8-1)(6\cdot29.747-1)$
8.		4	25	**Not Mersenne-number** $2^{25}-1=33.554.431=(6\cdot5+1)(6\cdot100+1)(6\cdot300+1)$
9.	5		29	$M_{29}=2^{29}-1=$**536.870.911**$=(6\cdot39-1)(6\cdot384.028-1)$
10.		5	31	$M_{31}=2^{31}-1=$**2.147.483.647 (prime)**
11.	6		35	**Not Mersenne-number** $2^{35}-1=34.359.738.367=(6\cdot5+1)(6\cdot12-1)(6\cdot21+1)(6\cdot20.487-1)$
12.		6	37	$M_{37}=2^{37}-1=$**137.438.953.471**$=(6\cdot37+1)(6\cdot102.719.696+1)$
13.	7		41	$M_{41}=2^{41}-1=$**2.199.023.255.551**$=(6\cdot2.228-1)(6\cdot27.418.559-1)$
14.		7	43	$M_{43}=2^{43}-1=$**8.796.093.022.207**$=(6\cdot698.148+1)(6\cdot349.977+1)$
15.	8		47	$M_{47}=2^{47}-1=$**140.737.488.355.327**$=(6\cdot392-1)(6\cdot9.977.136.563-1)$
16.		8	49	**Not Mersenne-number** $2^{49}-1=562.949.953.421.311=(6\cdot21+1)(6\cdot738.779.466.432+1)$
17.	9		53	$M_{53}=2^{53}-1=$**9.007.199.254.740.991**$=(6\cdot11.572-1)(6\cdot21.621.464.127-1)$
18.		9	55	**Not Mersenne-number** $2^{55}-1=36.028.797.018.963.967=(6\cdot4-1)(6\cdot5+1)(6\cdot15-1)(6\cdot147-1)$ $(6\cdot532-1)(6\cdot33.660+1)$
19.	10		59	$M_{59}=2^{59}-1=$**576.460.752.303.423.487 (prime)**
20.		10	61	$M_{61}=2^{61}-1=$**2.305.843.009.213.693.951 (prime)**
21.	11		65	**Not Mersenne-number** $2^{65}-1=36.893.488.147.419.103.231=(6\cdot5+1)(6\cdot1.365+1)$ $(6\cdot24.215.857.259.685+1)$
22.		11	67	$M_{67}=2^{67}-1=$**147.573.952.589.676.412.927**$=$ $(6\cdot32.284.620+1)(6\cdot126.973.042.881+1)$

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