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GRAPH THEORETICAL APROACH TO STRUCTURAL REPRESENTATION OF SYSTEMS

An attempt to generalize the holography principle

Introduction

This paper is designed to demonstrate how pattern recognition questions can be extended to general systems, or inversely, how structural identification of general systems can be traced back to pattern recognition question. For this purpose I'm suggesting one general way to project systems on to a mathematical model, called *sturctural representation* as in the headline of this paper. Subject to a suitable technical equipment, this act of representation is possible physically by means of image, sound, etc. With the further interesting outlook to recognize even such thing as social phenomena (systems).
This mathematical model can furthermore be regarded as *a generalized model of the holography principle* with reference to optical phenomena.

1. Necessity of the uniform reference system

The phenomena (systems) under study are which have been detached according to certain qualites from reality. Consequently, the same system can be studied from different sides, as the research purpose may require. These qualities (called herunder *variables*) lend themselves in a given case to characterizing the system. A variable, of course, can assume different values called *code-values*, each of which can be assigned to one element in the set of natural numbers.

1.1. Definition
Let X = {x_1, x_2, …, x_n} be a set of given variables which have sets K_1, K_2, …, K_n as code-sets, and let

(1.1) $$\mathfrak{E} = K_1 \times K_2 \times ... \times K_n$$

where × is the Descartes-multiplication. Then the sequence of *n* members e = (a_1, a_2, …, a_n) is called a *realization of the set X*, if e∈ $\mathfrak{E}$ namely

(1.2) $$a_1 \in K_1, a_2 \in K_2, ..., a_n \in K_n$$

1.2. Definition
The triad S = (X, E, R) is called a *system* if
X: the set of variables which define the system
E: the set of objects (elements), to each of which as they make up the system one realization of X can be assigned, namely

(1.3) $$\forall e \in E \rightarrow \exists!\, f \in \mathfrak{E}$$

R: the set of relations interpreted on the set E, namely the set of relations between the objects, namely

(1.4) $$\forall r \in R \Rightarrow \quad r \subseteq E \times E$$

The structure of an S = (X, E, R) system denotes the sum of relation structures from R. Each relation structure is domonstrated by a graph. The system is generally defined by several variables of different type. (These can be divided into the two groups of qualitative and quantitative variables.)
In every case, the joint manipulation of several variables of different type calls for a *common reference system*, since each variable reveals one aspect of the system. The problems of multivariate analysis are handled as a rule by methods of mathematical statistics and have the common feature to deal with transforming the variables (scale transformation, standardization, etc.) This means that the multidimensional space, definid by the variables, makes up the common reference system. Hence we encounter the following paradox:
All the information about a system is obtaind by measuring its elements (under no matter which variable), consequently the uniform basis of reference, as long as we collect information is obtained from the elements of the system, but will be won from the variables as soon as we describe it.

Owing to this paradox, in transforming the space of variables to a managable form we encounter a lot of problems which make it difficult to render an adequate description for the structure of the system. (See [1], [3], [4], [5] ,[6], [7]) Here, I should like to draw attention to a very interesting parallel.

2. The basic idea of the holography principle

According to the basic idea of the holography principle, the structural projection of objects (as special systems) necessitates a uniform reference system. D. Gábor when delivering a lecture in 1971. at his Nobel Prize award ceremony, said he was wondering why shouldn't it be possible first to accept the wrong electron image which however contains all the information, next to correct that wrong image by means of optical devices? Presently became clear to him such correction is possible, if at all, by a coherent electron beam. But on an ordinary photo the phases have got completely lost, as the intensity gets only recorded. ***No wonder, we lose the phase if there's nothing to compare with!***
Now, what happens if the light wave is given a coherent background in the form of a basic wave? The answer is rather easy: *„The object wave and the coherent background (or reference) wave generate interference strips in the places of phase identity."*
For this interference image D. Gábor applied the term *hologram, meaning the structural projection of the object*. (see [8], [9[1], [3])

3. Uniform reference system in case of general systems

As we saw in the paradox under Clause 1., the system – defining variables are not the carriers of elementary information, but are implements to measure them. The elementary carriers of a system (S) structure are the elements of the set E. This is also true because the system structure is determined by relations in the set R, which are interpreted on E. According to definition 1.2. precisely one realization of the set of variables X can be assigned to each element of the set E, which means that each arbitrary element (object) of the set E carriers exactly one elementary information (measurment) for all the variables. It follows that even in the case of a general system there exists a uniform reference system with regard to the variables, consisting of objects in the set E which carry the elementary information.

Consequently the sturcture of a system will be projected with the structure of the variables, as perceptible on the elementary information carriers, - and not with their concrete values.

4. Sturctural projection of systems

According to definition 1.2. take a system S = (X, E, R) and definie the relations $r_1', r_2', ..., r_n' \in R$ for variables X={x_1,x_2,…,x_n} in such a way that if e=($a_1a_2…a_n$)∈ E and f=($b_1b_2…b_n$) ∈ E then for any r_i∈ R we receive

(4.1) $$e\ r_i\ f \Leftrightarrow \quad a_i\ r_i'\ b_i \qquad (r_i' \subseteq K_i \times K_i)$$

Thus, to each x_i∈ X variable we have clearly assigned one r_i∈ R relation defined all on the same set (E). These are called *structure-generating relations* for the corresponding variables. Furthermore for each variable x_i∈ X we have assigned one well defined structure on the set E, represented on the following graph.
Mark the graph of the variable x_i with symbol $\Gamma_i = (P_i, H_i)$ where P_i is the set of the vertices and H_i is the set of the edges.

P_i: one vertex in P_i is assigned to each element of the set E, i.e. the elements of the two sets are assigned to each other according to a one-to-one correspondence (ρ tranformation), or if $e_j \in E,\ p_j \in P_i$ then

(4.2) $$\rho(e_j) = p_j$$

H_i: $H_i \subseteq P_i \times P$ more precisely

(4.3) $$(p_j p_k) \in H_i \quad \Leftrightarrow \quad e_j\ r_i\ e_k$$

As a superposition of the graphs Γ_i (i = 1,2…….,n) a multigraph projected from the system S is generated where the edges of individual structures of variables can be differentiated by colouring.
By this procedure the system S has been projected to a structure of relations between the objects in E, and the structure itself can be represented by the superposition of the graphs $\Gamma_1, \Gamma_2, ..., \Gamma_n$. The steps of the procedure are shown schematically in Figure 1.

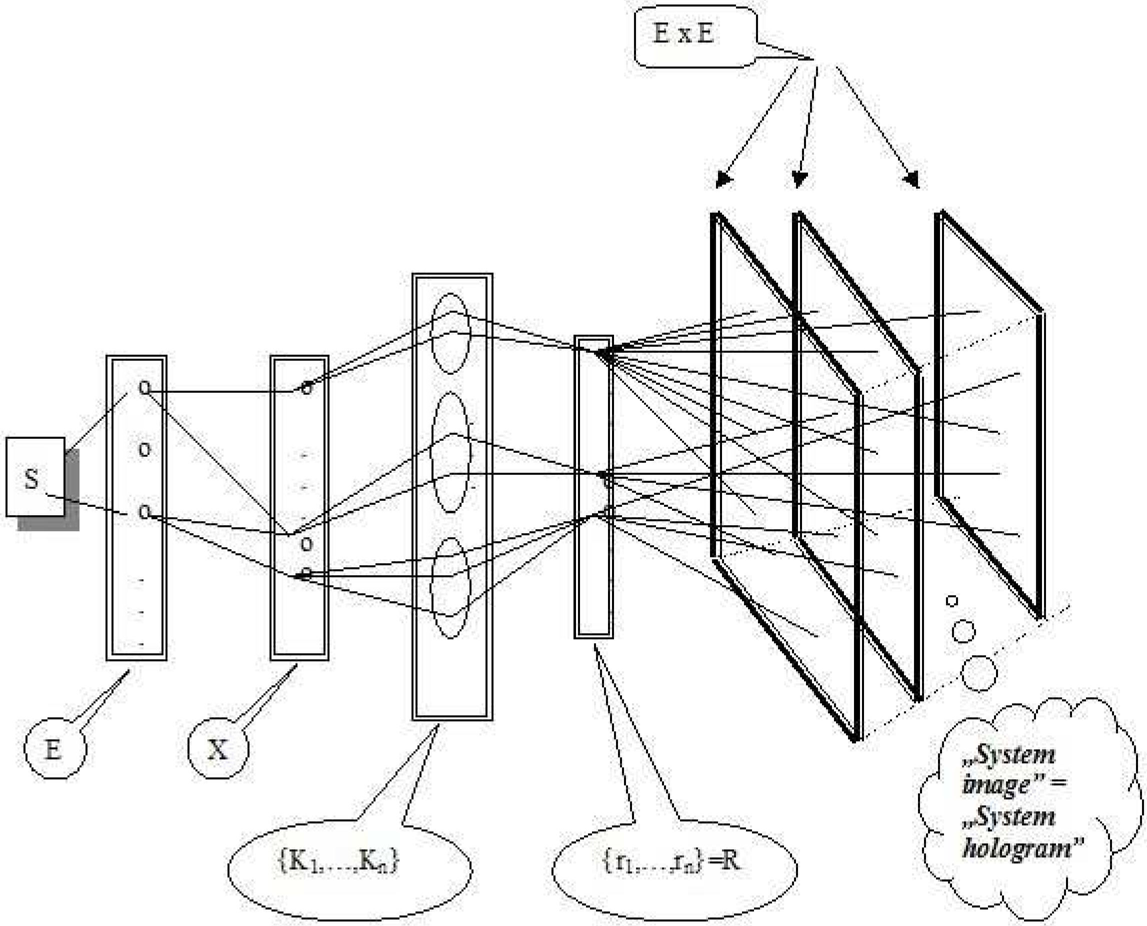

Figure 1.

One image matrix, as we know, can be assigned unambiguously to one graph (and vice versa) in such a way that if $M_{\Gamma} = [M_{ij}]^{mxm}$ (*m* is the number of row and column too) is the image matrix of the graph $\Gamma = (P, H)$ with $|P|$=m , where $|P|$ is the cardinality of set P, then

(4.4)
$$M_{ij} = \begin{cases} 1 & if \quad (p_i p_j) \in H \\ 0 & if \quad (p_i p_j) \notin H \end{cases}$$

Each element of the matrix M_{Γ} represents one image-point which is black or white according as the element value is 1 or 0.

Thus to each variable x_i we have assigned one image matrix I_{Γ_i} and if these are properly projected one on the other, we receive as a result the so-called *„system image"* to the superposed graphs.

This „system image" can also be visualized by physical means, showing what can be regarded as the abstract image of the system S. Now consider the holography discussed in Clause 2.

Every object can be conceived as a special system, defined for the purpose of visual demonstration, by four variables namely three coordinates of spatial extension as well as colour (intensity). The elements in the system are the object-points where the coherent reference waves are applied by holography as their *uniform reference system*.

If $\{R_1, R_2, R_3\}=R$ are the relations between the object-points, meeting the postulate as per (4.1), then the message of these relations will be: „the reference and the object waves of the given object-points are in the same phase” and the three are equivalent relations thus the interference strips are the physical manifestations of the equivalent relations and the hologram is a special „system image”.

As a final conclusion, we are in a position to study general systems upon uniform basis and again to employ methods of pattern recogrition in the examination of general systems (see [3]).

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