STRUCTURE-MEMORY (SM) as a universal graph model of the human memory

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1. **Introduction**

The memory is hardly separable part of the brain's activity, this is the reason of great challenge for the modelling. This paper is designed to demonstrate how fundamental psichologycal fact and important brain`s activity can be simulated in a uniform mathamatical model-system.
For this purpose I am suggesting one general way to project human memory on to a mathematical model, called STRUCTURE-MEMORY (SM) as in the headline of this paper.
As we know the human memory contains a lot of connections among the elements (for example: neurons) like the structure of system. For this structure and its functions modeling I use the graph theory.

2. **The brain's activities what are the objects of our modelling**

The fundamental activities of the human memory can be devided into the next three groups:
- the information reception and storing,
- the information recalling and
- the forgetting.

Here I should like to draw attention to a few basioc quality of these activities.

2.1. The informations have different sign-intensity when they make receive into the memory.
The sign-intensity depends on
- how important is the information for the person,
- what is the person's general condition,
- how many times repeated the same information?

2.2. The information storing in the human memory is one of the greatest problem of the modelling.
- This is the consequence of enormous quantity and variety of informations,
- and being unlimited connections of theirs (it is the basis of association).

2.3. The fundamental principle of information recalling is the association.
In our discussion we have two type of associations:
- the **similarity-association** is called a recalling if two or more informations recall according to their connections of texts,
- the **in time-association** is called a recalling if two or more information recall because they was storing at the same time.

From the point of wiew of the chances of recalling be of special importance the **retroactive inhibition**.

2.4. The **forgetting** is a generic term, which have more type:
- as a result of brain damage,
- as a result of brain's inactivity,
- as a result of retroactive inhibition,
- as a result of repression.

Over the mext time I should like to demonstrate a structural modelling method to simulate the psichologycal phenomena according to the above.

3. **The mathematical model of STRUCTURE-MEMORY (SM)**

The mathematical model build up to the graph theory. Before the mathematical modelling I have to describe an axiom:
To each informations (which we are storing in meory) we have clearly assigned one multi graph.

Denote the informations as I_1, I_2, …, I_j, …., the graph which assign to I_j information as G_j and the multi graph which assign to STRUCTURAL-MEMORY (in a shorter form: SM) as G_{SM} .
Let $G_{SM} =(P_{SM} ,E_{SM})$ where

(1) $P_{SM} = \{P_1, P_2, ..., P_n\}$

(2) $E_{SM} \subseteq P_{SM} \times P_{SM}$

namely P_{SM} is the set of the vertices and E_{SM} is the set of arcs („×" is the Cartesian prodact of two sets). G_{SM} is a multi graph with labelled vertices and colored arcs. Colour arcs of G_j graph assigned to I_j information the *"j colour"*, namely the *j label* are assigned to each arcs of G_j .
Let define the sum (denote: $\oplus$) of two $G_i =(P_i , E_i)$, $G_j =(P_j , E_j)$ graphs:

(3) $G_i \oplus G_j = (P_i \cup P_j \ , E_i \bigcup E_j)$

where $\bigcup$ is *extended-union* as I called, which definition is as follows:

(4) $E_i \bigcup E_j = (E_i \cup E_j) \cup (E_i \cap E_j)$

Thus $E_i \bigcup E_j$ set contaions the arcs of $E_i \cap E_j$ with twice multiplicity. If SM stored in a concrete time the I_1 , I_2 , …, I_k informations, then

(5) $G_{SM} = G_1 \oplus G_2 \oplus ... \oplus G_k$

namely

(6) $P_{SM} = P_1 \cup P_2 \cup ... \cup P_k$

(7) $E_{SM} = E_1 \bigcup E_2 \bigcup ... \bigcup E_k$

For example in the graph on Fig.1.: k=3, $P_{SM} = \{A, B, C, D, E, F\}$,

E_1 = E_2 = E_3 =

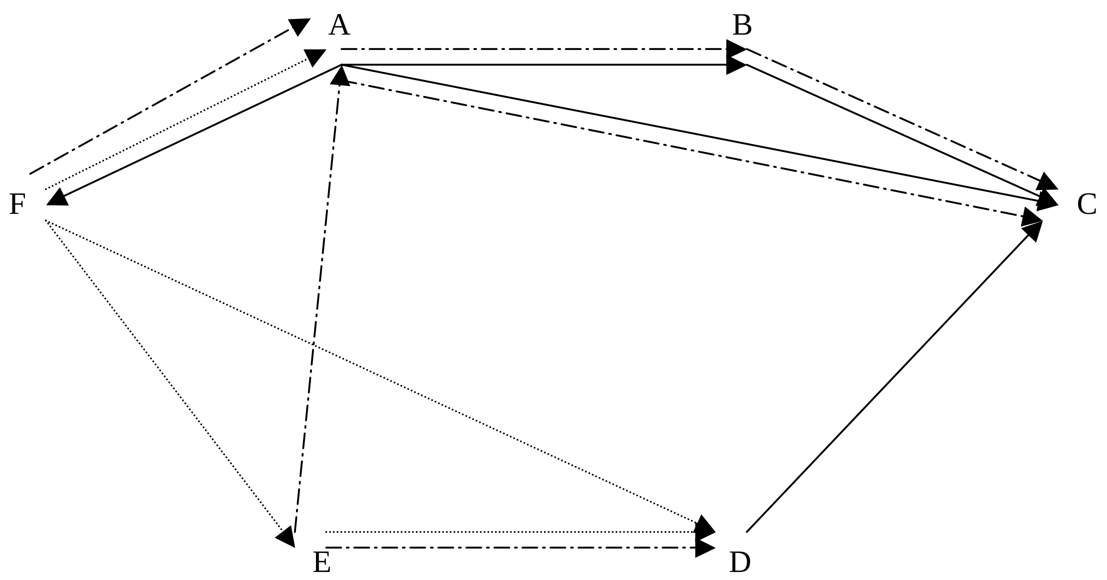

Fig. 1.

Easy to see, if $|P_{SM}| = n$ is the cardinality of set P_{SM} , then the maximum number of different information in SM is 2^{n^2}, because it is the number of different graphs on n vertices. If $k = 2^{n^2}$ is the number of the informations in SM, then

(8) $n = \sqrt{\log_2 k}$

So, if we take it that in every seconds 10^6 informations stored in SM until 200 years, then the number of graphs in G_{SM} (denote: k):

(9) $$k = 200 \cdot 10^6 \cdot 3600 \cdot 24 \cdot 352 = 10^{15} \cdot 6.08256$$

In this case according to (8) we get the next value for *n*:

(10) $$n = \sqrt{\log_2 (10^{15} \cdot 6.08256)} \approx 7.2411$$

namely the G_{SM} which represents an SM storer with (9) number of informations just need n=8 vertices.

4. **The activities of human memory described by the mathematical model**

Let denote the weight of $(p_u\, p_v)$ arc of G_i graph with symbol s^i_{uv} . Let the weight of $(p_u\, p_v)$ arc of G_{SM} graph be

(11) $$s^{SM}_{uv} = \begin{cases} \coprod_{i=1}^{k} s^i_{uv} & if \quad \forall s^i_{uv} \neq 0 \\ 0 & if \quad \exists 1 \leq i \leq k : s^i_{uv} = 0 \end{cases}$$

where *k* is the number of informations in SM at a concrete time.

4.1.
According to the quality under clause 2.1. in our model, the weight of arc has a quality and a structural component (denote these one with symbols α^i_{uv} , β^i_{uv}) for the case of s^i_{uv} weight.
Thus the criteriones under clause 2.1. means that the β-type weight component of arcs of any G_i graph has possible different values just at the time of storing. In addition I note that the colouring of arcs represents the time ordering.

4.2.
We now turn to the problem of association (see under clause 2.3.) and the information recalling. What is the problem of information recalling?
Given an input information I_j , which represents the G_j graph. Find an information (it represents G_i) with much similarity to G_j . Let see the association algorithm in our model. Consider the G_i graph which satisfy

(12) $$\forall G_r : \quad G_r \neq G_i \, , G_r \neq G_j \quad \Rightarrow \quad \left| E_j \cap E_i \right| \geq \left| E_j \cap E_r \right|$$

If we have more than one graphs with (12) property (let denote these G_i and G_t), then the associated graph G_j’ as follows:

(13) $$s_i = \sum_{(p_u p_r) \in E_i} s_{uv}^{SM}$$

(14) $$s_t = \sum_{(p_u p_r) \in E_t} s_{uv}^{SM}$$

(15) $$G_j^{'} = G_i \quad \Leftrightarrow \quad s_i \rangle\, s_t$$

If $s_i = s_t$, then

(16) $$G_j^{'} = G_i \quad \Leftrightarrow \quad t \langle\, i$$

The relation $t \langle\, i$ is everyting decidable, because *t*, *i* are the colouring of arcs, which represents the time of storing. Easy to see that the *in time-association* (see under clause 2.3.) is generalizing of (12)-(16) as we define

(17) $$I_r \in H_T \quad \Leftrightarrow \quad j - T \le r$$

where *j* is a given moment of the in time-association, *T* is a time period (j-T, j) and H_T is the set of informations which have stored in T time period. The summary of simulations of association in our model is shown in Fig. 2.

Association type	*Order of selecting*
similarity	(12) – (15) – (16)
in time	(17) – (15) – (12)

Fig. 2.

Now consider the simulate of retroactive inhibition. Let $G_j = (P_j, E_j)$ be a stimulating graph and $G_j' = (P_j', E_j')$ is associated to G_j . From the procedure of information's storing (see clause 3.), it follows that

(18) $$(p_u p_r) \in E_j \cap E_j^{'} \quad \Rightarrow \quad \left(s_{uv}^{SM}\right)_{j^{'}} \langle \left(s_{uv}^{SM}\right)_j$$

where $\left(s_{uv}^{SM}\right)_{j^{'}}$ and $\left(s_{uv}^{SM}\right)_j$ are the weights of $(p_u\, p_v)$ arc of G_{SM} as we defined in (11). According to the retroactive inhibition the process of changing on G_j and G_j' such that

(19) $$(p_u p_v) \in E_j \quad \Rightarrow \quad \left(s_{uv}^{j}\right)_j \langle \left(s_{uv}^{j}\right)_{j-\varepsilon}$$

(20) $(p_u p_v) \in E_j^{'} \quad \Rightarrow \quad \left(s_{uv}^{j^{'}}\right)_{j^{'}} \rangle\rangle \left(s_{uv}^{j^{'}}\right)_j$

where $\rangle\rangle$ denotes the "even less than" relation and $j-\varepsilon$ denotes the moment of a little bit before until j time.
From the (19), (20) statements we obtain the main characterisation of the phenomena:

(21) $E_j \cap E_j^{'} \to 0 \quad \Rightarrow \quad \left(s_{uv}^{j^{'}}\right)_j \to \left(s_{uv}^{j^{'}}\right)_{j^{'}}$ and $\left(s_{uv}^{j}\right)_j \to \left(s_{uv}^{j}\right)_{j-\varepsilon}$

We shall now study the properties of the repeat of information. Clearly if $I_i = I_j$ and $i \langle j$, then $G_i = G_j$ and according to (21) we obtain:

(22) $(p_u p_v) \in E_i \quad \Rightarrow \quad s_{uv}^{i} = 0$

(23) $(p_u p_v) \in E_j \quad \Rightarrow \quad \left(s_{uv}^{j}\right)_j \langle \left(s_{uv}^{j}\right)_{j-\varepsilon}$

and from (18) it follows that

(24) $(p_u p_v) \in (E_i \cap E_j) = E_j \quad \Rightarrow \quad \left(s_{uv}^{SM}\right)_i \langle \left(s_{uv}^{SM}\right)_j$

Here I should like to draw attention that if we increase the number of repeating, namely $I_1 = I_2 = \ldots = I_t$ and $t \to \infty$, then according to the (22), (23), (18) statements we obtain that:

(25) $(p_u p_v) \in E_r \quad \Rightarrow \quad s_{uv}^{r} = 0 \qquad (1 \le r \le t)$

(26) $(p_u p_v) \in E_t \quad \Rightarrow \quad s_{uv}^{t} \to 0$

(27) $(p_u p_v) \in E_t \quad \Rightarrow \quad s_{uv}^{SM} \to \infty$

Consequently the retroactive inhibition is mixing of the association and forgetting.

4.3.
We shall now discuss the problem of forgetting. Assign to SM an RG generator which makes random graphs on n vertices of G_{SM} such that every random graphs have same probability. Clearly, this probability is $\frac{1}{2^{n^2}}$ (see clause 3.).

Let Γ_z be a random graph which makes RG at the z moment and let the weight of each arcs of Γ_z be 1. If G_i is the associated graph to Γ_z (see clause 4.2.), then

(28) $(p_u p_v) \in E_i \quad and \quad \left(s_{uv}^{SM}\right)_i \neq 0 \quad \Rightarrow \quad \left(s_{uv}^{SM}\right)_z = \left(s_{uv}^{SM}\right)_i - 1$

Clearly from the (28), (19), (20) statements that it is possible the weight of some arcs of G_{SM} turn into zero, namely it means disappearance of these arcs. This property demonstrates that owing to forgetting (simulated with (28)), the structure of G_{SM} is changing.
So it is possible that G_{SM} be converted into disconnected, consequently the chains of associations be cut off. This consequence leads on to the field of thinking, but it is not the theme in this paper. As we shown by this property, that our model is more general just like the simulated of memory.

5. **Basic theorem of structures storing to SM**

I described my one axiom in start of clause 3. It is enable to assign a G_j multi graph to the I_j information. But the question is the following:
How should the G_j graph (which represents the I_j information) be stored to the SM ?

Definition 1.
Two graphs G_j and G_k are said to be isomorphic (let denote: $\cong$), if they have the same number of arcs and if there exists a bijection ρ such that

(29) $$G_j \cong G_k \quad \Rightarrow \quad \exists \rho : P_j \to P_k$$

(30) $$\forall p \in P_j \quad \Rightarrow \quad \exists! q \in P_k : \rho(p) = q$$

(31) $$(pp') \in E_j \quad \Leftrightarrow \quad (\rho(p)\rho(p')) \in E_k$$

Remark:
A special type of isomorphysm, if ρ is identical, namely

(32) $$\forall p_i \in P_j \quad \Rightarrow \quad \rho(p_i) = p_i \in P_k$$

Definition 2.
Let G_i and G_j be two arbitrary graphs and let $\rho : G_j \to G_j^{'} = \left(P_j^{'}, E_j^{'}\right)$ be a transformation in which case the $G_j^{'} \cap G_i$ is maximal, namely

(33) $$\rho : G_j \to G_j^{'} = \left(P_j^{'}, E_j^{'}\right) \Rightarrow \forall \rho^* : G_j \to G_j^* = \left(P_j^*, E_j^*\right), \rho^* \neq \rho \Rightarrow \left|E_j \cap E_j^{'}\right| \geq \left|E_j \cap E_j^*\right|$$

Then the isomorphic level of two graphs G_i and G_j is the cardinality of $E_j \cap E_j^{'}$ set (denote: $m(G_i, G_j)$), namely

(34) $$m(G_i, G_j) = \left|E_j \cap E_j^{'}\right|$$

Now let see the algorithm of the storing of G_j graph to SM (the denotes I use like in the definitions). Consider the ρ_u (u = 1,2,...,r) transformations. Let G_j is the input graph and G'_{SM} is the result of procedure, then

$$
\begin{aligned}
(35)\qquad & \rho_u(G_j) = G_j^{'} = (P_j^{'}, E_j^{'}) \Rightarrow \forall p_s \in P_j^{'} \cap P_{i_u} \Rightarrow \\
& \Rightarrow \exists!\, p_t \in P_j : \rho_u(p_t) = p_s \Rightarrow t' = s,\ \mathit{otherwise}\ t' = t
\end{aligned}
$$

$$
(36)\qquad G_{SM}^{'} = G_{SM} \oplus G_{i_u}
$$

The following is the main theorem of the structures storing to SM:
Let G_j and G_i be two graphs. If $G_j \cong G_i$, then G_j and G_i are storing to SM with equivalent labels of vertices.

I draw ahead of the proof of this theorem owing to lack of space. But I have some final remarks to the consequences of this theorem.

Remarks:

1. The consequence of the theorem we obtain that in SM come into being the functional blocks. This property of the brain is well-know in the medical science.
2. From the storing method (see (35), (36)) implies too, that in SM come into being overlapping functional blocks. According to this property we can modeling the storing of associations among the informations at the time of storing.

Finaly I note that in this paper my aim is to demonstrate how brain’s activity can be simulated by the SM. Furthermore the graph theory as the mathematical model of SM is well-fitted to modeling this one.