THE CONSTRUCTION OF MATHEMATICS

THE HUMAN MIND'S GREATEST ACHIEVEMENT

KLAUS TRUEMPER

"I am fully convinced that, based on some very elementary and not yet understood endowment of our brain, the fantastic mathematical universe is human-made. This can't be proved mathematically. The best one can hope for are compelling arguments and strong empirical evidence. This is what Klaus Truemper's book 'The Construction of Mathematics: The Human Mind's Greatest Achievement' delivers."
– Martin Grötschel, mathematician and President, Berlin-Brandenburg Academy of Sciences and Humanities, Germany

"Klaus Truemper has made an original and daring attack on the foundations of mathematics. Readers will enjoy his forthright and unswerving analysis. His ideas should become recognized and influential."
– Reuben Hersh, mathematician and award-winning author of a number of books on the nature, practice, and social impact of mathematics

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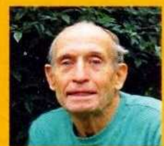
Is mathematics created or discovered? The answer has been debated for centuries. This book answers the question clearly and decisively by applying the concept of language games, invented by the philosopher Wittgenstein to solve difficult philosophical issues.

Using the results of modern brain science, the book also explains how it is possible that eminent mathematicians and scientists offer diametrically opposed answers to the question of creation vs. discovery.

Interested in the topic but intimidated by mathematics? Not to worry. If you are familiar with the elementary operations of addition, subtraction, multiplication, and division, you can follow the arguments of this book.



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The hyperbolic points are the points of *D* that do not lie on the boundary of *D*, such as point *P*.

The hyperbolic lines are represented by arcs of circles that lie within *D* and make a right angle with the boundary of *D*, such as lines *B* and *C*, or by lines that are diameters of *D*, such as line *A*.

Lines *B* and *C* go through the point *P* and do not intersect with the line *A*. According to the definition of "parallel," the lines *B* and *C* are thus parallel to line *A*.[257]

The display preserves angles, and by inspection we can tell that example triangle *T* has angles summing to less than 180 degrees.

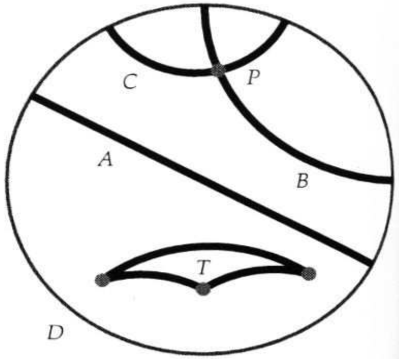
This is entirely different from Euclidean geometry, which can be displayed with straight lines and with exactly one parallel line for any given point outside a given line.

Lobachevsky published his results in Russian; mathematicians outside Russia became aware of his work only years later.

In 1832, János Bolyai (1802–1860) also developed the hyperbolic geometry, again independently.

When Gauss became aware of Bolyai's result, he declared that he had thought of this result decades ago,[259] but also

Jules Henri Poincaré.[255]



Poincaré's disk *D*: *A*, *B*, and *C* are lines. *T* is a triangle.[256]

János Bolyai.[258]

are an infinite number of parallel lines going through that point.

258. Source: `http://www.titoktan.hu/Bolyai_a.htm`. "János Bolyai" photo by copyright holder Támas Dénes, who has kindly granted permission to use the photo. According to [Dénes, 2011] and the Wikipedia "János Bolyai" entry, no original portrait of Bolyai survives, and an unauthentic picture appears in some encyclopedias and on a Hungarian postage stamp.
The relief of János Bolyai shown here is part of six reliefs in front of the Culture Palace in Marosvásárhely, Romania. According to the investigation reported in [Dénes, 2011] that included computer simulation using images across three generations, the relief likely is a good, indeed the only authentic, representation of the mathematician.

259. See Wikipedia "Non-Euclidean geometry."

260. See Wikipedia "János Bolyai."

261. Gauss's evaluation is confirmed by recent investigations into the work of János Bolyai; see [Dénes, 2011] and [Kiss, 1999].

[Dénes, 2011] Dénes, T. (2011). Real Face of János Bolyai. *Notices of the American Mathematical Society*, Jan. 2011.