Application of the Dénes type Symmetric Prime Number theorem to proof of *there exist infinitely many primes of the form n^2+1*

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Abstract

Due to the *Dénes type Symmetric Prime Number theorem* in [Dénes 2017], we prove that *there exist infinitely many primes of the form n^2+1.*

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Definition 1. *(Symmetrical prime pair)*

Let $N \geq 4$ and $0 \leq m_N \leq N/2$ be natural numbers. If $p_{N-} = N - m_N$ and $p_{N+} = N + m_N$ are prime numbers, then these are called *symmetric prime pair for N.*

Theorem 1.

If p and q are arbitrary two prime numbers, then exist N and m_N natural numbers, *that p and q are symmetric prime pair for N.*

PROOF

Due to the 1st Theorem in [Dénes 2001], if p and q are primes then they has the forms $p=6k\pm1$ and $q=6r\pm1$ (k and r are natural numbers). Thus the following cases are possible, where the value of $N = \frac{p+q}{2}$ are always integer:

(1) $$N = \frac{p+q}{2} = \frac{(6k+1)+(6r+1)}{2} = \frac{6(k+r)+2}{2} = 3(k+r)+1 \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

(2) $$N = \frac{p+q}{2} = \frac{(6k+1)+(6r-1)}{2} = \frac{6(k+r)}{2} = 3(k+r) \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

(3) $$N = \frac{p+q}{2} = \frac{(6k-1)+(6r+1)}{2} = \frac{6(k+r)}{2} = 3(k+r) \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

(4) $$N = \frac{p+q}{2} = \frac{(6k-1)+(6r-1)}{2} = \frac{6(k+r)-2}{2} = 3(k+r)-1 \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

Select the following m_N values to calculated N in (1) - (4):

(5) $$m_N = \frac{|p-q|}{2} = \frac{|(6k+1)-(6r+1)|}{2} = \frac{6|(k-r)|}{2} = 3|(k-r)| \quad (k=1,2,\ldots),\ (r=1,2,\ldots)$$

(6) $$m_N = \frac{|p-q|}{2} = \frac{|(6k+1)-(6r-1)|}{2} = \frac{6|(k-r)|+2}{2} = 3|(k-r)|+1 \quad (k=1,2,\ldots),\ (r=1,2,\ldots)$$

(7) $$m_N = \frac{|p-q|}{2} = \frac{|(6k-1)-(6r+1)|}{2} = \frac{6|(k-r)|-2}{2} = 3|(k-r)|-1 \quad (k=1,2,\ldots),\ (r=1,2,\ldots)$$

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(8) $m_N = \frac{|p-q|}{2} = \frac{|(6k-1)-(6r-1)|}{2} = \frac{6|(k-r)|}{2} = 3|(k-r)|$ $(k=1,2,\ldots)$, $(r=1,2,\ldots)$

As it follows from (1)-(4) and (5)-(8):

(9) $$\begin{aligned} N - m_N &\overset{(1),(5)}{=} 3(k+r)+1-3(k-r) = 6r+1 = q \\ N + m_N &\overset{(1),(5)}{=} 3(k+r)+1+3(k-r) = 6k+1 = p \end{aligned} \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

(10) $$\begin{aligned} N - m_N &\overset{(2),(6)}{=} 3(k+r)-(3(k-r)+1) = 6r-1 = q \\ N + m_N &\overset{(2),(6)}{=} 3(k+r)+(3(k-r)+1) = 6k+1 = p \end{aligned} \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

(11) $$\begin{aligned} N - m_N &\overset{(3),(7)}{=} 3(k+r)-(3(k-r)-1) = 6r+1 = q \\ N + m_N &\overset{(3),(7)}{=} 3(k+r)+(3(k-r)-1) = 6k-1 = p \end{aligned} \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

(12) $$\begin{aligned} N - m_N &\overset{(4),(8)}{=} 3(k+r)-1-3(k-r) = 6r-1 = q \\ N + m_N &\overset{(4),(8)}{=} 3(k+r)-1+3(k-r) = 6k-1 = p \end{aligned} \quad (k=1,2,3,\ldots),\ (r=1,2,3,\ldots)$$

The connections (9)-(12) correspond exactly to the *Dénes type Symmetric Prime Number theorem* in [Dénes 2017].

Q.E.D.

The consequence of the Theorem 1. is the following Theorem 2.:

Theorem 2.

For arbitrary p prime number there exist N and m_N natural numbers, that $p=N-m_N$ and *$q=N+m_N$ is prime.*

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For the following 3th Theorem we use the following known connection:

(13) $$\sum_{i=1}^{n}(2i-1) = 2\left(\sum_{i=1}^{n} i\right) - n = \frac{2n(n-1)}{2} - n = 1+3+5+\ldots+(2n-3)+(2n-1) = n^2$$

It follows that

(14) $$\frac{2n(n-1)-2n}{2} + 1 = 2n\cdot\frac{n}{2} + 1 = 1+3+5+\ldots+(2n-3)+2n = n^2+1$$

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Theorem 3.

There exist infinitely many primes of the form n^2+1.

PROOF (indirect)

Suppose that N is the last natural number for which $p=N^2+1$ is the prime number.
p is a prime number this implies that N^2+1 is odd, so it is sufficient to prove only for even $n>N$. Then the indirect condition can be written as follows:

(15) $\forall c \geq 1 \quad \Rightarrow \quad q = (\underbrace{N+2c}_{n})^2 + 1$ ***not prime number*** (c is an integer)

(16) $q = (\underbrace{N+2c}_{n})^2 + 1 = \underbrace{N^2+1}_{p} + 4Nc + 4c^2 = p + 4c(N+c)$

It follows from Theorem 2. that there is a natural number c to which it is true

(17) $m_p = 2c(N+c) \quad \Rightarrow \quad q = p + 2m_p \overset{(16)}{=} (\underbrace{N+2c}_{n \rangle N})^2 + 1$ ***prime number*** (See Figure 1.)

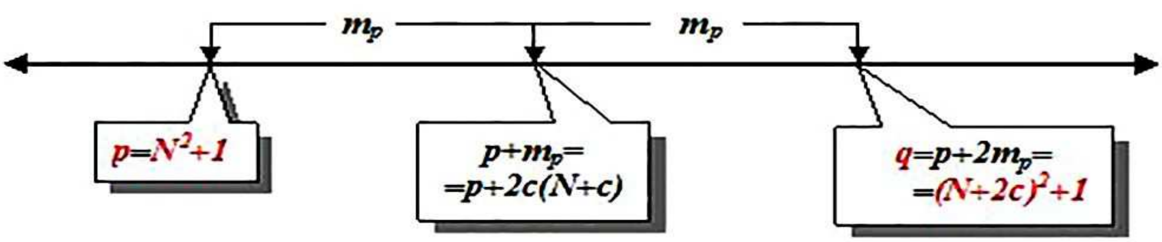

Figure 1.

However, this contradicts the indirect condition (15).

Q.E.D.

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The calculation of the first ten and the first few more than 1 million prime numbers of the form n^2+1 are demonstrated in Table 1.

Table 1.

N	$p=N^2+1$	c	$m_p=2c(N+c)$	$q=p+2m_p$	$q=(N+2c)^2+1$	Symmetr. primes
2	**5** prime	**1**	***6***	17	**17** prime	*
4	**17** prime	**1**	***10***	37	**37** prime	*
6	37 prime	1	14	65	65=5x13	
6	**37** prime	**2**	***32***	101	**101** prime	*
10	101 prime	1	22	145	145=5x29	
10	**101** prime	**2**	***48***	197	**197** prime	*
14	**197** prime	**1**	***30***	257	**257** prime	*
16	257 prime	1	34	325	$325=5^2$x13	
16	**257** prime	**2**	***72***	401	**401** prime	*
20	401 prime	1	42	485	485=5x97	
20	**401** prime	**2**	***88***	577	**577** prime	*
24	**577** prime	**1**	***50***	677	**677** prime	*
26	677 prime	1	54	785	785=5x157	
26	677 prime	2	112	901	901=17x53	
26	677 prime	3	174	1.025	$1.025=5^2$x41	
26	677 prime	4	240	1.157	1.157=13x89	
26	**677** prime	**5**	***310***	1.297	**1.297** prime	*
36	1.297 prime	1	74	1.445	$1.445=5\text{x}17^2$	
36	**1.297** prime	**2**	***152***	1.601	**1.601** prime	*
. . .						
1.004	1.008.017 prime	1	2.010	1.012.037	1.012.037=13x77.849	
1.004	1.008.017 prime	2	4.024	1.016.065	1.016.065=5x203.213	
1.004	**1.008.017** prime	**3**	***6.042***	1.020.101	**1.020.101** prime	*
1.010	1.020.101 prime	1	2.022	1.024.145	1.024.145=5x257x797	
1.010	1.020.101 prime	2	4.048	1.028.197	1.028.197=109x9.433	
1.010	1.020.101 prime	3	6.078	1.032.257	1.032.257=17x41x1.481	
1.010	1.020.101 prime	4	8.112	1.036.325	$1.036.325=5^2$x41.453	
1.010	1.020.101 prime	5	10.150	1.040.401	1.040.401=101x10.301	
1.010	1.020.101 prime	6	12.192	1.044.485	1.044.485=5x13x16.069	
1.010	1.020.101 prime	7	14.238	1.048.577	1.048.577=17x61.681	
1.010	1.020.101 prime	8	16.288	1.052.677	1.052.677=61x17.257	
1.010	1.020.101 prime	9	18.342	1.056.785	1.056.785=5x241x877	
1.010	1.020.101 prime	10	20.400	1.060.901	1.060.901=37x53x541	
1.010	1.020.101 prime	11	22.462	1.065.025	$1.065.025=5^2$x13x29x113	
1.010	1.020.101 prime	12	24.528	1.069.157	1.069.157=41x89x293	
1.010	**1.020.101** prime	**13**	***26.598***	1.073.297	**1.073.297** prime	*
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N	$p=N^2+1$	*c*	$m_p=2c(N+c)$	$q=p+2m_p$	$q=(N+2c)^2+1$	Symmetr. primes
3.624	13.133.377 prime	1	7.250	13.147.877	13.147.877=101x349x373	
3.624	13.133.377 prime	2	14.504	13.162.385	13.162.385=5x2.632.477	
3.624	13.133.377 prime	3	21.762	13.176.901	13.176.901=109x120.889	
3.624	13.133.377 prime	4	29.024	13.191.425	13.191.425=5x13x37x1.097	
3.624	13.133.377 prime	5	36.290	13.205.957	13.205.957=17x53x14.657	
3.624	13.133.377 prime	6	43.560	13.220.497	13.220.497=397x33.301	
3.624	13.133.377 prime	7	50.834	13.235.045	13.235.045=5x2.647.009	
3.624	13.133.377 prime	8	58.112	13.249.601	13.249.601=41x461x701	
3.624	13.133.377 prime	9	65.394	13.264.165	13.264.165=5x17x29x5.381	
3.624	**13.133.377** prime	**10**	***72.680***	13.278.737	**13.278.737** prime	*
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3.894	15.163.237 prime	1	7.790	15.178.817	15.178.817=73x337x617	
3.894	15.163.237 prime	2	15.584	15.194.405	15.194.405=5x29x104.789	
3.894	**15.163.237** prime	**3**	***23.382***	15.210.001	**15.210.001** prime	*
3.900	15.210.001 prime	1	7.802	15.225.605	15.225.605=5x97x31.393	
3.900	15.210.001 prime	2	15.608	15.241.217	15.241.217=41x371.737	
3.900	15.210.001 prime	3	23.418	15.256.837	15.256.837=17x897.461	
3.900	15.210.001 prime	4	31.232	15.272.465	15.272.465=5x13x234.961	
3.900	**15.210.001** prime	**5**	***39.050***	15.288.101	**15.288.101** prime	*

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