Proof of the Twin prime conjecture

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Abstract

The author has been proven in 2001 the *complementary prime sieve theorem* (see [Dénes 2001]). Due to this theorem, for any *N=6k+1* type natural number are composite if and only if one of the following is fulfilled: *k=6uv+u+v* or *k=6uv-u-v*.
Based on this theorem is the indirect proof of the Twin prime conjecture, so thereafter we can called *Twin prime theorem*.

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Twin prime theorem

There are infinite many *p, p+2* type prime pairs.

Proof (indirect)

Below Table 1. lists all the prime numbers in its first and third columns (see Theorem 1. in [Dénes 2001]). Suppose, that in the *Kth* line is the last twin pair, that is

(1a)
$$6K-1,\,6K+1 \text{ are primes and } \forall k = K+x \quad (x \text{ natural number}) \Rightarrow$$
$$\Rightarrow \quad \text{If } 6k-1 \text{ is prime, then } 6k+1 \text{ is NOT PRIME}$$

or

(1b)
$$6K-1,\,6K+1 \text{ are primes and } \forall k = K+x \quad (x \text{ natural number}) \Rightarrow$$
$$\Rightarrow \quad \text{If } 6k+1 \text{ is prime, then } 6k-1 \text{ is NOT PRIME}$$

For the (1a) point due to the Theorem 2. in [Dénes 2001], for *k=K+x* one of the following is fulfilled:

(2) $k=6uv+u+v$ ($u>0$ and $v>0$ natural numbers)

or

(3) $k=6uv-u-v$ ($u>0$ and $v>0$ natural numbers)

From the indirect condition (1a) and from (2) we get the following connections:

(4)
$$k=K+x=6uv+u+v \Rightarrow 6K+1=6(6uv+u+v-x)+1=\underbrace{6(6uv+u+v)+1}_{(1a) \text{ and } (2) \Rightarrow NOT\ PRIME}-6x$$

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From the indirect condition (1a) and from (3) we get the following connections:

(5) $$k=K+x=6uv-u-v \Rightarrow 6K+1=6(6uv-u-v-x)+1=\underbrace{6(6uv-u-v)+1}_{(1a)\ and\ (3)\Rightarrow NOT\ PRIME}-6x$$

(4) or (5) must be satisfied for every $x>0$ natural number. Thus, it is sufficient to prove that there are infinite numbers of x which do not satisfy equation (4) and (5). In other words, the right side of the equations are NOT PRIME for infinite many x.

Suppose, that in (4) is $u=v$ then the the next is true:

(6) $$6(6uv+u+v)+1\overset{u=v}{=}6(6u^2+2u)+1=36u^2+12u+1=(6u+1)^2\overset{(4)}{\Rightarrow}$$
$$\overset{(4)}{\Rightarrow}\ 6K+1=(6u+1)^2-6x$$

Also be $x=6^{2z+1}$, where $z>0$ natural number, then from (6) we get the following:

(7) $$6K+1=(6u+1)^2-6x=(6u+1)^2-6\cdot 6^{2z+1}=(6u+1)^2-\left(6^{z+1}\right)^2=$$
$$=\left(6u+1+6^{z+1}\right)\left(6u+1-6^{z+1}\right)$$

We get a similar result if we assume that in (5) is $u=v$:

(8) $$6(6uv-u-v)+1\overset{u=v}{=}6(6u^2-2u)+1=36u^2-12u+1=(6u-1)^2\overset{(4)}{\Rightarrow}$$
$$\overset{(4)}{\Rightarrow}\ 6K+1=(6u-1)^2-6x$$

Also be $x=6^{2z+1}$, where $z>0$ natural number, then from (8) we get the following:

(9) $$6K+1=(6u-1)^2-6x=(6u-1)^2-6\cdot 6^{2z+1}=(6u-1)^2-\left(6^{z+1}\right)^2=$$
$$=\left(6u-1+6^{z+1}\right)\left(6u-1-6^{z+1}\right)$$

(7) and (9) contradicts the indirect condition (1a), according to which the $6K+1$ is prime number.

In case (1b), the course of proof is similar.
That is for the (1b) point due to the Theorem 2. in [Dénes 2001], for $k=K+x$ one of the following is fulfilled:

(10) $k=6uv+u-v$ ($u>0$ and $v>0$ natural numbers)

or

(11) $k=6uv-u+v$ ($u>0$ and $v>0$ natural numbers)

Since (10) and (11) are symmetrical for u and v, it is sufficient to carry out the proof only (10). Then it comes from the indirect condition (1b) and (10) that

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(12) $$k=K+x=6uv+u-v \Rightarrow 6K-1=6(6uv+u-v-x)-1=\underbrace{6(6uv+u-v)-1}_{(1b)\ and\ (10)\Rightarrow NOT\ PRIME}-6x$$

(12) must be satisfied for every $x>0$ natural number. Thus, it is sufficient to prove that there are infinite numbers of x which do not satisfy equation (12). In other words, the right side of the equation are NOT PRIME for infinite many x.

Suppose, that in (12) is $u=v$ then the the next is true:

(13) $$6(6uv+u-v)-1\overset{u=v}{=}36u^2-1=(6u-1)(6u+1)\overset{(12)}{\Rightarrow}$$
$$\overset{(12)}{\Rightarrow}\quad 6K-1=(6u-1)(6u+1)-6x$$

Also be $6x=6^z(6u-1)$, where $z>0$ natural number, then from (13) we get the following:

(14) $$6K-1=(6u-1)(6u+1)-6^z(6u-1)=(6u-1)(6u+1-6^z)$$

(14) is satisfied for every u and z whenever

(15) $$6u+1\rangle 6^z \quad\Rightarrow\quad u\rangle\frac{6^z-1}{6}=\frac{5(6^{z-1}+6^{z-2}+...+6+1)}{6}$$

(15) is satisfied in each case if

(16) $$u=(6^{z-1}+6^{z-2}+...+6+1)\quad\Rightarrow\quad 6x=6^z\left(6(6^{z-1}+6^{z-2}+...+6+1)-1\right)\quad\Rightarrow$$
$$\Rightarrow\quad x=6^{z-1}\left((6^z+6^{z-1}+...+6^2+6)-1\right)$$

(14)-(16) contradicts the indirect condition (1b), according to which the *6K-1* is prime number.

Q.E.D.

References

[Dénes 2001] Dénes, Tamás: Complementary prime-sieve, PUre Mathematics and Applications, Vol.12 (2001), No. 2, pp. 197-207
http://www.titoktan.hu/_raktar/_e_vilagi_gondolatok/PUMA-CPS.pdf

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Table 1.

k	*6k-1* ↓	*6k*	*6k+1* ↓	*6k+2*	*6k+3*	*6k+4*
0			1	2	3	4
1	***5***	6	***7***	8	9	10
2	***11***	12	***13***	14	15	16
3	***17***	18	***19***	20	21	22
4	***23***	24	25	26	27	28
5	***29***	30	***31***	32	33	34
6	35	36	***37***	38	39	40
7	***41***	42	***43***	44	45	46
8	***47***	48	49	50	51	52
9	***53***	54	55	56	57	58
10	***59***	60	***61***	62	63	64
11	65	66	***67***	68	69	70
12	***71***	72	***73***	74	75	76
13	77	78	***79***	80	81	82
14	***83***	84	85	86	87	88
15	***89***	90	91	92	93	94
16	95	96	***97***	98	99	100
17	***101***	102	***103***	104	105	106
18	***107***	108	***109***	110	111	112
19	***113***	114	115	116	117	118
20	119	120	121	122	123	124
21	125	126	127	128	129	130
22	***131***	132	133	134	135	136
23	***137***	138	***139***	140	141	142
24	143	144	145	146	147	148
25	***149***	150	***151***	152	153	154
26	155	156	157	158	159	160
27	161	162	***163***	164	165	166
28	***167***	168	169	170	171	172
29	***173***	174	175	176	177	178
30	***179***	180	***181***	182	183	184
31	185	186	187	188	189	190

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Table 1. continue

k	*6k-1* ↓	*6k*	*6k+1* ↓	*6k+2*	*6k+3*	*6k+4*
32	***191***	192	***193***	194	195	196
33	***197***	198	***199***	200	201	202
…	…	…	…	…	…	…
137	***821***	822	***823***	824	825	826
138	***827***	828	***829***	830	831	832
…	…	…	…	…	…	…
247	***1.481***	1.482	***1.483***	1.484	1.485	1.486
248	***1.487***	1.488	***1.489***	1.490	1.491	1.492
…	…	…	…	…	…	…
312	***1.871***	1.872	***1.873***	1.874	1.875	1.876
313	***1.877***	1.878	***1.879***	1.880	1.881	1.882
…	…	…	…	…	…	…
347	***2.081***	2.082	***2.083***	2.084	2.085	2.086
348	***2.087***	2.088	***2.089***	2.090	2.091	2.092
…	…	…	…	…	…	…
542	***3.251***	3.252	***3.253***	3.254	3.255	3.256
543	***3.257***	3.258	***3.259***	3.260	3.261	3.262
…	…	…	…	…	…	…
577	***3.461***	3.462	***3.463***	3.464	3.465	3.466
578	***3.467***	3.468	***3.469***	3.470	3.471	3.472
…	…	…	…	…	…	…
942	***5.651***	5.652	***5.653***	5.654	5.655	5.656
943	***5.657***	5.658	***5.659***	5.660	5.661	5.662
…	…	…	…	…	…	…
1.572	***9.431***	9.432	***9.433***	9.434	9.435	9.436
1.573	***9.437***	9.438	***9.439***	9.440	9.441	9.442
…	…	…	…	…	…	…
3.502	***21.011***	21.012	***21.013***	21.014	21.015	21.016
3.503	***21.017***	21.018	***21.019***	21.020	21.021	21.022
…	…	…	…	…	…	…
975.237	***5.851.421***	*5.851.422*	***5.851.423***	*5.851.424*	*5.851.425*	*5.851.426*
975.238	***5.851.427***	*5.851.428*	***5.851.429***	*5.851.430*	*5.851.431*	*5.851.432*
…	…	…	…	…	…	…
K	***6K-1***		***6K+1***			
…	…	…	…	…	…	…
k=K+x	*6k-1=6(K+x)-1*		*6k+1=6(K+x)+1*			